DOCUMENT RESUME

ED 408 175 SE 060 263

AUTHOR	Coutts, Linda; And Others
TITLE	Missouri's Framework for Curriculum Development in Mathematics K-12.
INSTITUTION	Missouri State Dept. of Elementary and Secondary Education, Jefferson City.
PUB DATE	96
NOTE	200p.
AVAILABLE FROM	Missouri Dept. of Elementary and Secondary Education, P.O. Box 480, Jefferson City, MO 65102-0480.
PUB TYPE	Guides - Non-Classroom (055)
EDRS PRICE	EDRS Price MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Classroom Communication; *Curriculum Development; *Educational Strategies; Elementary Secondary Education; Integrated Curriculum; Interdisciplinary Approach; Mathematics Education; *Mathematics Instruction; Mathematics Skills; Problem Solving; State Curriculum Guides; *State Standards; Teaching Methods

ABSTRACT

Missouri's Frameworks for Curriculum Development in Mathematics acknowledges that teachers will bring the vision, ideals, and principles of the Show-Me Standards into their classrooms in exciting and innovative ways. The role of the frameworks is to provide districts with a "frame" for building curricula using the standards as a foundation. The frameworks provide indicators of what students should know and be able to do by the end of grades 4, 8, and 12. The frameworks contain suggested resources, discussions of issues and current practices, and examples of quality student work. Many of the suggested activities encourage an integrated, interdisciplinary curriculum. The final section provides a discussion of how districts might begin to explore the advantages of curriculum integration. The strands include: (1) problem solving; (2) communication; (3) reasoning; (4) connections; (5) number sense; (6) geometric and spatial sense; (7) data analysis, probability, and statistics; (8) patterns and relationships; (9) mathematical systems and number theory; and (10) discrete mathematics. The section on suggested practices for teaching and learning mathematics contains topics such as basic knowledge, technology, connections and active learning, problem solving, varied instructional methods, communication and reasoning, classroom climate, and assessment. Contains 12 references. (JRH)

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Framework for Curriculum Development In Mathematics K-12

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Further Acknowledgments:

Missouri's Frameworks for Curriculum Development have been reviewed by over 200 district Professional Development Committees, members of professional organizations of the various disciplines, vocational and special education teachers, members of the State Board of Education, and board members of the Missouri School Boards Association and the Missouri Congress of PTA. Subsequently, many of the suggestions offered by these reviewers have been incorporated into the Frameworks.

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PREFACE

In recent years, people from all walks of American life have participated in wide-ranging discussions about the direction of American education. While the situation is not as dire as some critics would have us believe, there is clearly a need for the review of current school organization, curricula, teaching practices and academic standards for students. As a result of these discussions, many legislators, educators, parents, and business leaders across the nation have initiated projects designed to develop high academic standards in safe learning environments for all students. The Show-Me State is no exception.

A Vision for Missouri Schools

In charting a path for the future of Missouri public schools, a common vision of what that future looks like is an essential first step. These schools must be physically and intellectually safe places for students. In these schools we believe all students are capable of learning and are challenged by high expectations. To meet this challenge, students must have a solid foundation of basic knowledge and skills. In addition, they must also be able to apply what they know.

To support the development of such students, teachers actively engage them in exploring, analyzing and understanding the world. Learning is interesting, hands-on and has real applications that are important to students. We recognize that students learn better, remember more, and develop deeper understandings of both knowledge and thinking processes if they have the opportunity to actively explore content and concepts. They view learning as the on-going process of extending and fine tuning their ideas and understanding through application. As a result, they take responsibility for their own learning, see its worth and make connections to the world beyond the classroom.

In addition, teachers and administrators in these communities see themselves as learners. They gain greater insight into their profession from their experiences among their students and in contact with their colleagues. They constantly seek to extend their knowledge and expertise, and frequently engage in professional development activities to deepen their professional skills and understanding.

The schools we envision also involve parents, business leaders and interested citizens in a multitude of educational activities. In turn, these people accept the responsibility for supporting and nurturing the learners within the school and see themselves as an important part of the learning community.

The Outstanding Schools Act

The passage of the Outstanding Schools Act in 1993 signaled Missouri's commitment to a public school system that purposefully prepares young people for the 21st century and assures our state's continued economic vitality. The Show-Me State has issued an ambitious, common-sense agenda by setting challenging academic standards for all students, by supporting professional development of educators to improve the quality of curriculum and instruction, and by providing more equitable funding for public education. In addition, the Outstanding Schools Act calls for increased accountability in improving student academic performance for all of Missouri's public school districts and school buildings. In summary, it addresses the following initiatives:

The Show-Me Standards–a set of 73 rigorous standards–intended to define what students should know and be able to do by the time they graduate from Missouri's public high schools;

Curriculum Frameworks–frameworks for curriculum development in communication arts, fine arts, health and physical education, mathematics, science, social studies and curriculum integration–intended to provide assistance to districts in aligning local curriculum with the Show-Me Standards;

A New Statewide Assessment–a new assessment system of performance events and multiple choice and short answer questions–intended to provide an indication of how well students are meeting the Show-Me Standards and how well they compare academically with other students across the nation;

Professional Development for Educators–one percent of the local district's basic state aid and one percent of the state educational budget to be set aside to support professional development–intended to improve student performance through improving the performance of educators; and

Professional Standards for New Educators–professional standards defining what graduating pre-service teachers should know and be able to do as certificated Missouri teachers–intended to ensure that there is a strong link between teacher preparation and the expectations of the school community.

Strategies for Including All Students

We believe virtually all students are able to learn, even though all students are not succeeding in school. Many of them fall behind and leave school before graduation. These strategies help all students become more engaged in learning:

- Use a multisensory approach to teaching and learning, including auditory, visual and hands-on techniques
- Present concepts in several ways, linking them to what students already know and checking frequently for understanding
- Model learning strategies and encourage students to talk about their own thinking and learning processes
- Teach students how to organize their thoughts using drawings, charts, outlines, thumbnail sketches, and computer generated flowcharts, spread sheets, and databases, etc.
- Encourage students to use technology as a user-friendly tool for learning
- Use questions and approaches that require inquiry, problem solving and the synthesizing of ideas
- Provide equal opportunities for all students to participate in class activities
- Adapt materials to accommodate students with special needs
- Provide real-life and work applications of what students should know and be able to do

Technology in Missouri Education

The Outstanding Schools Act also provides funding through technology grants for districts to obtain and access the latest technologies. The Show-Me Standards emphasize understanding and use of technology as a tool for learning, and *Missouri's Frameworks for Curriculum Development* suggest many ways to incorporate the use of technology in the classroom. Because students learn and demonstrate knowledge in a variety of ways, technologies can help teachers accommodate the learning and presentation styles of each student, keep students on task, provide individualized assistance and support students who must be absent from class.

Technology also plays a major role in adequately preparing students for continuous learning and for the workplace. More households are becoming dependent on technology (such as subscribing to on-line banking and informational services and operating computers, VCRs, CD-ROMs and voice mail). Growing numbers of jobs require an understanding or use of technology. In fact, we cannot imagine the impact technology will have on the lives of our children and grandchildren.

An important outgrowth of technology in our schools is that it may also enhance the local economy by encouraging schools and communities to share technologies and training. Through the Outstanding Schools Act, our students and other community members will have opportunities to become technologically literate.

The Missouri School Improvement Program

The Missouri School Improvement Program (MSIP) provides additional support to these measures by requiring districts to have a long-range plan for ongoing curriculum development and revision, to develop written curriculum guides for all curricular areas, and to implement the stated curriculum. In the second cycle of reviews (1996-2001), MSIP teams will evaluate whether a district's written curriculum addresses the Show-Me Standards. Curriculum guides for at least one content area must be revised to reflect the Show-Me Standards one year after the adoption of *Missouri's Frameworks for Curriculum Development* by the State Board of Education. Guides for the six content areas contained in the frameworks must be revised to address the Show-Me Standards by the start of the 2000-2001 school year. In planning for curriculum development and revision, districts may wish to consider the Department's schedule for implementation of the new assessment system:

Mathematics	Spring 1997
Communication Arts	Spring 1998
Science	Spring 1998
Social Studies	Spring 1999
Health/Physical Education	Spring 2000
Fine Arts	Spring 2000

The first administrations of the mathematics, communication arts, and science assessments will be voluntary. Districts will be required to administer the new mathematics assessment in 1998 and the new communication arts and science assessments in 1999. At the time each test is required, the *Missouri Mastery and Achievement Test* (MMAT) will no longer be used to collect

state achievement data. The remaining subject areas will be required as noted in the above list. Important curricular areas not included in the state assessment, such as foreign languages and practical arts, will find that the Show-Me Standards encompass knowledge and skills that are applicable for their students as well. In fact, the Show-Me Standards should be integrated throughout the entire K-12 curriculum so that all high school graduates will be better prepared to meet the challenges of the 21st century.

The Show-Me Standards

The Outstanding Schools Act requires that the State Board of Education oversee the development of "not more than 75 academic performance standards." The Show-Me Standards, developed over a period of two years with input from teachers, school officials and citizens, are the result of this mandate. There are **40 *knowledge (content) standards*** that provide a solid foundation of knowledge in communication arts, fine arts, health and physical education, mathematics, science, and social studies.

Business and higher education communities have pointed out that, in general, students are graduating with some factual knowledge, but they are not skilled in abstract thinking, problem solving, and working cooperatively or collaboratively. Students need practice in integrating, applying and transferring what they are learning in one context or content area to new and different situations. To remedy this, the Missouri teachers who developed the Show-Me Standards proposed **33 *performance (process) standards*** . These standards include important process skills that students should master in order to successfully gather, analyze and apply information and ideas; communicate effectively within and beyond the classroom; recognize and solve problems; and make decisions and act as responsible members of society. (The Show-Me Standards follow this Preface.)

While intended to establish higher expectations for **all** of Missouri's students, the 73 Show-Me Standards do not represent everything a student should or will learn. Graduates who meet these standards, however, should be well-prepared for further education, work, and civic responsibilities.

The Department of Elementary and Secondary Education believes that the preservation of local control is a hallmark of the Outstanding Schools Act. Local school districts have the authority, the ability and the resources to develop rigorous and challenging curriculum that will prepare their students to be successful in the 21st century. The Department's role is to support districts in this endeavor, helping them carry out this task by offering technical assistance, professional development opportunities, and new technologies. Each school district must determine how its curriculum will be structured and which methods to use to implement that curriculum in the classroom. We believe that local educators, parents, employers, and community leaders know best how to incorporate the Show-Me Standards into their districts' curricula to meet the needs of their students.

Missouri's Frameworks for Curriculum Development

Missouri's Frameworks for Curriculum Development in communication arts, fine arts, health and physical education, mathematics, science, social studies, and curriculum integration acknowledge that teachers will bring the vision, ideals and principles of the Show-Me Standards into their classrooms in exciting and innovative ways. The role of the frameworks is to provide districts with a "frame" for building curricula using the Show-Me Standards as a foundation. District curriculum guides furnish the interior curriculum plan and appropriate instruction. District guides probably are not organized in the same way as the six content frameworks. The good news is that they do not have to be. In fact, although the frameworks use a three-column format ("What All Students Should Know," "What All Students Should Be Able to Do," and "Sample Learning Activities"), the committees who wrote the frameworks approached their task in ways that best appeared to fit their particular content areas. For example, the social studies framework is organized around important questions; communication arts, around the four goals of the Show-Me Standards; and science, around the knowledge standards. Each group of framework writers had valid reasons for their organizational plan, just as curriculum developers in each district will and should. Similarly, local curriculum guides should be in a format that is the most useful for the district's teachers.

Curriculum Frameworks Are Resources–

- To assist districts in developing curriculum consistent with the Show-Me Standards
- To indicate what students should know and be able to do by the end of grades 4, 8 and 12.
- To support teachers in providing high-quality learning experiences for all students
- To suggest examples of hands-on, real-world activities and classroom performance assessments
- To suggest effective learning and teaching strategies supported by research and practice
- To initiate discussions concerning curriculum integration within and across classrooms
- To provide examples of quality student work requiring knowledge and application of that knowledge

Curriculum Frameworks Are Not–

- Not required by law for district use
- Not detailed lesson plans or curricula
- Not items on which all students must be tested
- Not directives for uniform programs or textbook adoption
- Not mandates for inclusion of specific teaching methods or programs
- Not a format that all district curriculum guides must follow

As the writers worked on developing the frameworks, they asked themselves questions that are very similar to what local curriculum developers must ask: How do we see the Show-Me Standards playing out in this content area and across the curriculum? Are there important areas addressed by the standards that are not included in our current curriculum and how should we address these? Are we presenting a balanced curriculum, K-12? Are we giving students opportunities to apply in real situations the knowledge they have learned? How will the emphasis on what students know and are able to do change instruction, or will it? Do our assessments model and support the taught curriculum? In essence, the framework writers have tried to make the work of local curriculum committees easier by establishing a relationship between the Show-Me Standards and the K-12 curriculum in the six content areas. For some districts this step is unnecessary; for others, it will help give much needed direction.

In addition, the frameworks provide indicators of what students should know and be able to do by the end of grades 4, 8 and 12. They contain suggested resources, discussions of issues and current practices, and examples of quality student work. Many of the suggested activities encourage an integrated, interdisciplinary curriculum. In fact, the final section of *Missouri's Frameworks for Curriculum Development* provides a discussion of how districts might begin to explore the advantages of curriculum integration.

Meeting the Challenge

We know that, in most cases, the level of expectation and the interest of the students determine the level of performance. When schools set high standards and provide access to meaningful curriculum for every learner, students and teachers flourish. Success for all students is not a dream but a reality.

With the advent of the 21st century, we are on the threshold of more life-changing events than we can ever imagine. Computers and technology are making our lives both more productive and more complex. The Internet, fax machines, fiber-optic networks, and voice mail are changing the ways we communicate and do business. Information on every topic is readily available at the push of a button. Two bread-winners per family are the norm rather than the exception. Workers can no longer select a particular career and expect to do the same job in the same way year after year. As citizens, we must be prepared to make important decisions which will affect the lives of future generations in positive ways. The challenge we face now is how to educate our children to be successful as individuals and as members of society in a world that most of their grandparents would not even recognize. Working together, we can do it. What an exciting time to be involved in the education of Missouri's students!

THE SHOW-ME STANDARDS

Authority for the Show-Me Standards: Section 160.514, Revised Statutes of Missouri, and the Code of State Regulations, 5 CSR 50-375.100

PERFORMANCE (PROCESS) STANDARDS

GOAL 1: Students in Missouri public schools will acquire the knowledge and skills to gather, analyze and apply information and ideas.

Students will demonstrate within and integrate across all content areas the ability to

1.1 develop questions and ideas to initiate and refine research
1.2 conduct research to answer questions and evaluate information and ideas
1.3 design and conduct field and laboratory investigations to study nature and society
1.4 use technological tools and other resources to locate, select and organize information
1.5 comprehend and evaluate written, visual and oral presentations and works
1.6 discover and evaluate patterns and relationships in information, ideas and structures
1.7 evaluate the accuracy of information and the reliability of its sources
1.8 organize data, information and ideas into useful forms (including charts, graphs, outlines) for analysis or presentation
1.9 identify, analyze and compare the institutions, traditions and art forms of past and present societies
1.10 apply acquired information, ideas and skills to different contexts as students, workers, citizens and consumers

GOAL 2: Students in Missouri public schools will acquire the knowledge and skills to communicate effectively within and beyond the classroom.

Students will demonstrate within and integrate across all content areas the ability to

2.1 plan and make written, oral and visual presentations for a variety of purposes and audiences
2.2 review and revise communications to improve accuracy and clarity
2.3 exchange information, questions and ideas while recognizing the perspectives of others
2.4 present perceptions and ideas regarding works of the arts, humanities and sciences
2.5 perform or produce works in the fine and practical arts
2.6 apply communication techniques to the job search and to the workplace
2.7 use technological tools to exchange information and ideas

GOAL 3: Students in Missouri public schools will acquire the knowledge and skills to recognize and solve problems.

Students will demonstrate within and integrate across all content areas the ability to

3.1 identify problems and define their scope and elements
3.2 develop and apply strategies based on ways others have prevented or solved problems
3.3 develop and apply strategies based on one's own experience in preventing or solving problems
3.4 evaluate the processes used in recognizing and solving problems
3.5 reason inductively from a set of specific facts and deductively from general premises
3.6 examine problems and proposed solutions from multiple perspectives
3.7 evaluate the extent to which a strategy addresses the problem
3.8 assess costs, benefits and other consequences of proposed solutions

GOAL 4: Students in Missouri public schools will acquire the knowledge and skills to make decisions and act as responsible members of society.

Students will demonstrate within and integrate across all content areas the ability to

4.1 explain reasoning and identify information used to support decisions
4.2 understand and apply the rights and responsibilities of citizenship in Missouri and the United States
4.3 analyze the duties and responsibilities of individuals in societies
4.4 recognize and practice honesty and integrity in academic work and in the workplace
4.5 develop, monitor and revise plans of action to meet deadlines and accomplish goals
4.6 identify tasks that require a coordinated effort and work with others to complete those tasks
4.7 identify and apply practices that preserve and enhance the safety and health of self and others
4.8 explore, prepare for and seek educational and job opportunities

KNOWLEDGE (CONTENT) STANDARDS

Communication Arts

In Communication Arts, students in Missouri public schools will acquire a solid foundation that includes knowledge of and proficiency in

CA 1 speaking and writing standard English (including grammar, usage, punctuation, spelling, capitalization)
CA 2 reading and evaluating fiction, poetry and drama
CA 3 reading and evaluating nonfiction works and material (such as biographies, newspapers, technical manuals)
CA 4 writing formally (such as reports, narratives, essays) and informally (such as outlines, notes)
CA 5 comprehending and evaluating the content and artistic aspects of oral and visual presentations (such as storytelling, debates, lectures, multimedia productions)
CA 6 participating in formal and informal presentations and discussions of issues and ideas
CA 7 identifying and evaluating relationships between language and culture

Fine Arts

In Fine Arts, students in Missouri public schools will acquire a solid foundation that includes knowledge of

FA 1 process and techniques for the production, exhibition or performance of one or more of the visual or performed arts
FA 2 the principles and elements of different art forms
FA 3 the vocabulary to explain perceptions about and evaluations of works in dance, music, theater and visual arts
FA 4 interrelationships of visual and performing arts and the relationships of the arts to other disciplines
FA 5 visual and performing arts in historical and cultural contexts

Health/Physical Education

In Health/Physical Education, students in Missouri public schools will acquire a solid foundation that includes knowledge of

HP 1 structures of, functions of, and relationships among human body systems
HP 2 principles and practices of physical and mental health (such as personal health habits, nutrition, stress management)
HP 3 diseases and methods for prevention, treatment and control
HP 4 principles of movement and physical fitness
HP 5 methods used to assess health, reduce risk factors, and avoid high risk behaviors (such as violence, tobacco, alcohol and other drug use)
HP 6 consumer health issues (such as the effects of mass media and technologies on safety and health)
HP 7 responses to emergency situations

Mathematics

In Mathematics, students in Missouri public schools will acquire a solid foundation that includes knowledge of

MA 1 addition, subtraction, multiplication and division; other number sense, including numeration and estimation; and the application of these operations and concepts in the workplace and other situations
MA 2 geometric and spatial sense involving measurement (including length, area, volume), trigonometry, and similarity and transformations of shapes
MA 3 data analysis, probability and statistics
MA 4 patterns and relationships within and among functions and algebraic, geometric and trigonometric concepts
MA 5 mathematical systems (including real numbers, whole numbers, integers, fractions), geometry, and number theory (including primes, factors, multiples)
MA 6 discrete mathematics (such as graph theory, counting techniques, matrices)

Science

In Science, students in Missouri public schools will acquire a solid foundation that includes knowledge of

SC 1 properties and principles of matter and energy
SC 2 properties and principles of force and motion
SC 3 characteristics and interactions of living organisms
SC 4 changes in ecosystems and interactions of organisms with their environments
SC 5 processes (such as plate movement, water cycle, airflow), and interactions of Earth's biosphere, atmosphere, lithosphere, and hydrosphere
SC 6 composition and structure of the universe and the motions of the objects within it
SC 7 processes of scientific inquiry (such as formulating and testing hypotheses)
SC 8 impact of science, technology, and human activity on resources and the environment

Social Studies

In Social Studies, students in Missouri public schools will acquire a solid foundation that includes knowledge of

SS 1 principles expressed in the documents shaping constitutional democracy in the United States
SS 2 continuity and change in the history of Missouri, the United States and the world
SS 3 principles and processes of governance systems
SS 4 economic concepts (including productivity and the market system) and principles (including the laws of supply and demand)
SS 5 the major elements of geographical study and analysis (such as location, place, movement, regions) and their relationships to changes in society and environment
SS 6 relationships of the individual and groups to institutions and cultural traditions
SS 7 the use of tools of social science inquiry (such as surveys, statistics, maps, documents)

OVERVIEW OF MATHEMATICS EDUCATION

RATIONALE FOR THE STUDY OF MATHEMATICS

Changes in the workplace, in the media, and in information technology have radically changed the needs of all people for mathematics. Workers across Missouri and nationwide are involved in analyzing information, solving problems and making decisions.

"A major problem that we face currently is the vast gap between what many schools emphasize in defining success and what the world outside schools emphasizes in defining success." (Creating a Climate for Change...MATH, 1994) Society expects schools to produce "mathematically literate" workers. The notion of "minimum mathematics for the majority, advanced mathematics for a few" is no longer acceptable for students' future success. Today and into the future, "quality mathematics education for all students is essential for a healthy economy." (Everybody Counts, 1989) In order to prepare mathematically literate workers it is necessary for schools to develop students who learn to value mathematics, become confident in their own ability to do mathematics, become problem solvers, and are able to reason and make connections within and across the disciplines as well as to real-world situations. (NCTM, 1989)

World expectations for students include "making decisions, solving problems, finding the best solution, working on a team, using appropriate technology, creating new skills, and using education as a pump." (Creating a Climate for Change...MATH, 1994) Mathematics must be taught in a way to serve as a way to encourage students to pursue the mathematics necessary not only for their career choices but also for their personal lives and for becoming informed citizens. "Our highly technological society requires all students to be prepared to use mathematics to defend their rights." (Kappan, Feb. 1994) With changes in technology, in employment and in personal needs, citizens are responsible for their own learning. Schools must teach students how to be responsible for their own learning.

The ability of students to solve non-routine problems and understand concepts from a wide range of mathematical topics, such as algebra, geometry, and trigonometry, is vital to prepare students for their future. "Most jobs now require analytical rather than merely mechanical skills, so most students need more mathematical power in school." (Reshaping School Mathematics, 1990).

PURPOSE OF THIS FRAMEWORK

Missouri's Framework for Curriculum Development in Mathematics is intended to provide policymakers, instructional leaders, teachers, and community members with the support, definition, and direction necessary to reconstruct mathematics education in the school setting. It is not a curriculum; it does, however, provide the support needed by those educators who wish to generate and implement a new vision for how mathematics can be taught and learned in schools.

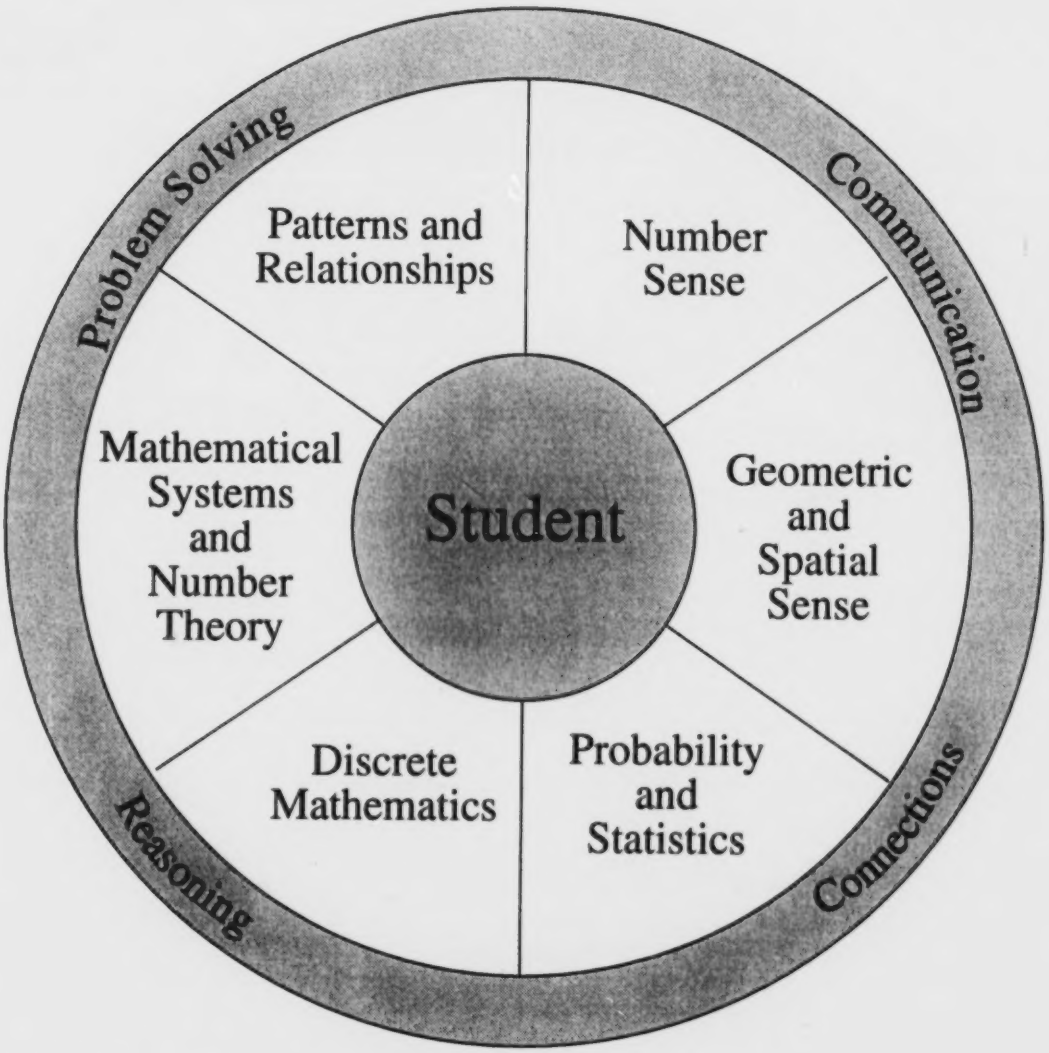
A goal of the Framework is to prepare students to become mathematically literate citizens. In order to meet this goal, students must learn to value mathematics, become confident in their ability to do mathematics, become proficient problem-solvers, and be able to make mathematical connections within and across disciplines as well as to the real world.

The Framework reflects what students must know and be able to do in order to succeed in mathematics. Students must be proficient in all mathematical content strands: number sense, geometric and spatial sense, data analysis and probability, patterns and relationships, mathematical systems and number theory, and discrete mathematics. They must also be able to use computers and calculators as tools that help solve problems.

This document builds on the *"Curriculum and Evaluation Standards for School Mathematics,"* published in 1989 by the National Council of Teachers of Mathematics. The processes of mathematics (problem-solving, communication, reasoning, and connections) are interwoven throughout all the content strands at all grade levels while the content strands spiral through the grade levels An overlap of strands is necessary so that students will develop and experience the interrelatedness of all areas of mathematics. Course titles and the development of the curriculum by course and/or grade levels are left to the local district. The scope and sequence of this framework is deliberately flexible so local districts can develop their own benchmarks and customize a scope and sequence that best suits their students.

The vision that is presented in the Show-Me Standards and the *Missouri's Framework for Curriculum Development in Mathematics* is articulated in high standards. All students can achieve these standards if an environment is created in which learning is both possible and expected. At the same time, attention to *all students* must not diminish the dedication to providing full encouragement and opportunity to explore mathematics in greater breadth and depth to those students who have interest or talent in pursuing careers that require additional mathematical achievement.

MAJOR ORGANIZING STRANDS



The strands in *Missouri's Framework for Curriculum Development in Mathematics* are closely tied to the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*. The middle column of the strands contains references to the NCTM Standards as well as to Missouri's Show-Me Standards.

The first four strands (problem solving, communication, reasoning, and connections) are the performance (process) strands. They define major areas of emphasis with respect to the teaching of mathematics and ultimate goals for all students of mathematics. The other six strands (number sense, geometric and spatial sense, data analysis/probability and statistics, patterns and relationships, mathematical systems and number theory, and discrete mathemat-

ics) are commonly known as the knowledge (content) strands. The six content strands define the mathematical knowledge that is to be taught using the four process strands. The NCTM process strands are closely related to the four goals of the Show-Me Standards: gather and apply information, communicate effectively, solve problems, and make decisions.

Performance (Process) Strands

Problem Solving

In order to make sound decisions, students must be able to recognize and pose problems as well as develop skills for investigating possible solutions. The central focus of the mathematics curriculum should be problem solving. The curriculum must provide students with the opportunity to solve relevant problems.

Communication

Through reading, writing, listening, viewing, speaking, researching and reporting, students can understand mathematics and use it effectively. Communication is important in mathematics because it creates a universal understanding of symbols. The effective use of communication techniques is necessary for multiple workplace and daily situations.

Reasoning

Reasoning is a process basic to all disciplines. It allows students to understand that mathematics is a way of thinking. Students should be encouraged to justify their conjectures and solutions. They must understand "that being able to explain and justify their thinking is important and that how a problem is solved is as important as its answer." (NCTM, 1989)

Connections

The need for all students to incorporate prior knowledge with real-world situations and integrate concepts across the curricular areas is vital. By focusing on the relationships between mathematical ideas and modeling, students are able to understand the powerful role mathematics plays in other disciplines. Interaction within and among mathematics and other disciplines makes the study of mathematics relevant. Mathematical connections allow construction of bridges between the concrete and the abstract, linking conceptual and procedural knowledge. These bridges invite students to informally explore, conjecture, and develop mathematical generalizations.

Knowledge (Content) Strands

Number Sense

The operations of addition, subtraction, multiplication and division as well as numeration and estimation are included in the number sense strand. Number sense incorporates the applications of these operations and concepts in the workplace and other situations. Through number sense, it is possible to gain a solid understanding of mathematics in the real world. Understanding numbers allows students to select the type of answer needed and a method necessary to arrive at a solution.

Geometric and Spatial Sense

Geometric and spatial relationships involve measurement including length, area, volume, trigonometric relationships, similarity and transformations of shapes. Geometry involves the study of visual patterns used in representing and describing the world in which we live. Spatial sense is needed to interpret, understand, and appreciate our geometric world.

Data Analysis, Probability and Statistics

Collecting, organizing and interpreting data allow students to make informed decisions based on information gathered and analyzed. Knowledge of probability allows students to investigate the likelihood that a particular event will occur. The study of probability enables students to make predictions based on experimental data.

Patterns and Relationships

Recognizing patterns and relationships has been instrumental in the development of mathematics and the study of numbers. Exploring patterns and relationships allows students to develop and understand how mathematics applies to their understanding of the development of connections between and among numbers, algebra, geometry, trigonometry, functions, and change relationships.

Mathematical Systems and Number Theory

Mathematical systems encompass the real number system, both rational numbers (whole numbers, integers, fractions, and decimals) and irrational numbers. Geometry may also be developed as a mathematical system. Number theory includes odd and even numbers, primes, factors and multiples. An understanding of mathematical systems and number theory allows students to see mathematics as a cohesive whole.

Discrete Mathematics

Discrete mathematics is the study of points, ideas, and objects that are separate from each other or distinct. Graph theory, counting techniques, matrices and the mathematics of decision making are included as part of discrete mathematics. Discrete mathematics promotes the making of mathematical connections, provides advantage of the technological setting, and provides the opportunities for critical thinking and mathematical reasoning.

ORGANIZATIONAL "ROAD MAP"

Number of Strand

Name of Strand

MATHEMATICS K-12

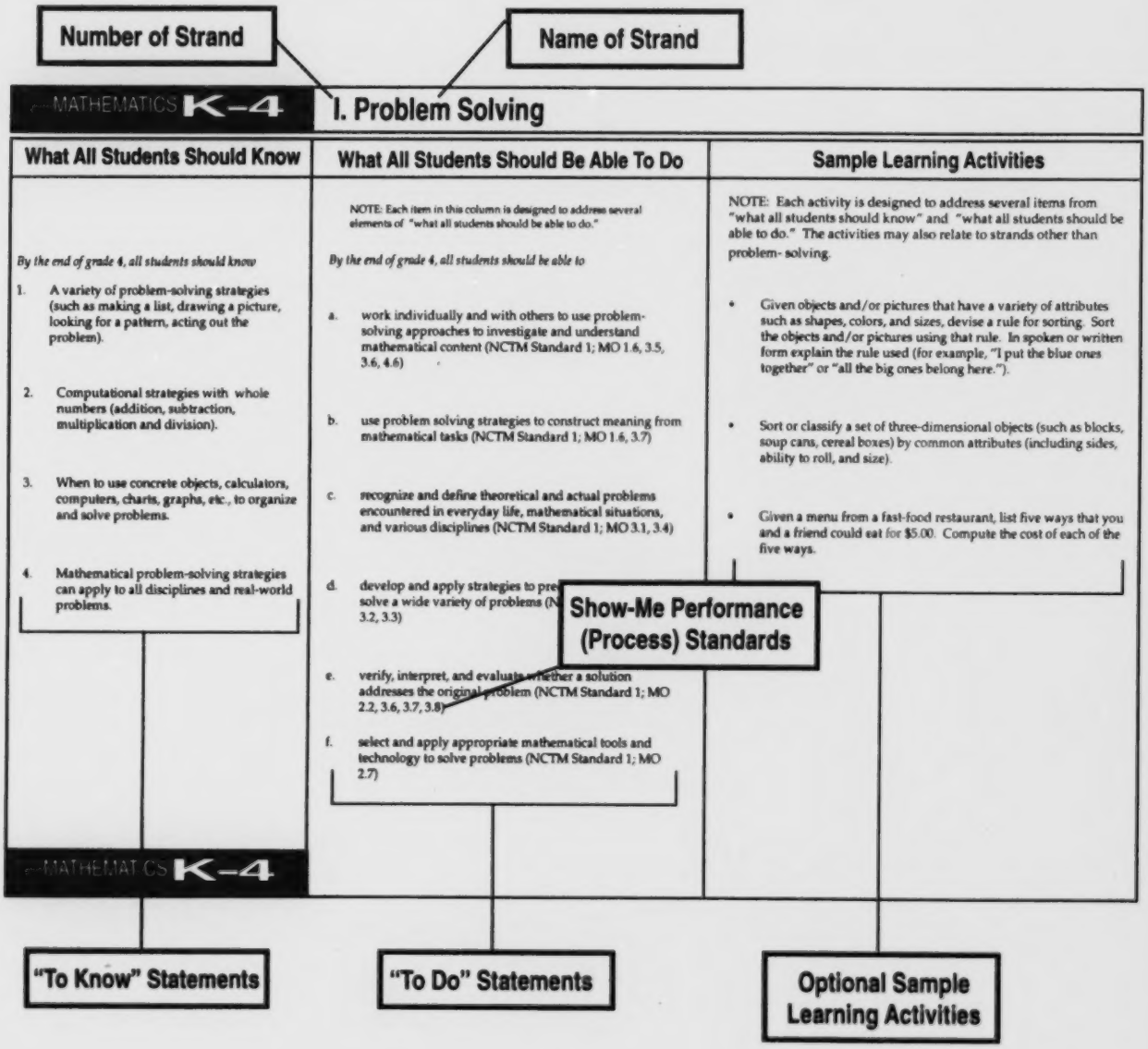
I. Problem Solving

K-12 Content Overview

Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematics activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned. Posing and solving problems should be the major focus of all students' mathematical activity so that through working with interesting, engaging, and intellectually stimulating situations, they come to understand mathematics and use it effectively.

Experiences should be such that students use discovery-oriented, inquiry-based and problem-centered approaches to investigate and understand mathematics. Students should be able to recognize, formulate, clarify and engage in solving problems arising from mathematical situations, everyday experiences, applications to other disciplines, and real-world applications.

MATHEMATICS K-12


Number of Strand
Name of Strand
MATHEMATICS K–4
I. Problem Solving
What All Students Should Know
What All Students Should Be Able To Do
Sample Learning Activities
NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."
NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem- solving.
By the end of grade 4, all students should know
By the end of grade 4, all students should be able to
1. A variety of problem-solving strategies (such as making a list, drawing a picture, looking for a pattern, acting out the problem).
2. Computational strategies with whole numbers (addition, subtraction, multiplication and division).
3. When to use concrete objects, calculators, computers, charts, graphs, etc., to organize and solve problems.
4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems.
a. work individually and with others to use problem-solving approaches to investigate and understand mathematical content (NCTM Standard 1; MO 1.6, 3.5, 3.6, 4.6)
b. use problem solving strategies to construct meaning from mathematical tasks (NCTM Standard 1; MO 1.6, 3.7)
c. recognize and define theoretical and actual problems encountered in everyday life, mathematical situations, and various disciplines (NCTM Standard 1; MO 3.1, 3.4)
d. develop and apply strategies to pre
solve a wide variety of problems (N
3.2, 3.3)
e. verify, interpret, and evaluate whether a solution addresses the original problem (NCTM Standard 1; MO 2.2, 3.6, 3.7, 3.8)
f. select and apply appropriate mathematical tools and technology to solve problems (NCTM Standard 1; MO 2.7)
• Given objects and/or pictures that have a variety of attributes such as shapes, colors, and sizes, devise a rule for sorting. Sort the objects and/or pictures using that rule. In spoken or written form explain the rule used (for example, "I put the blue ones together" or "all the big ones belong here.").
• Sort or classify a set of three-dimensional objects (such as blocks, soup cans, cereal boxes) by common attributes (including sides, ability to roll, and size).
• Given a menu from a fast-food restaurant, list five ways that you and a friend could eat for $5.00. Compute the cost of each of the five ways.
Show-Me Performance (Process) Standards
MATHEMATICS K–4
"To Know" Statements
"To Do" Statements
Optional Sample Learning Activities

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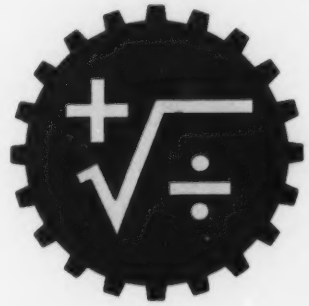
I. Problem Solving



K-12 Content Overview

"Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematics activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned." (NCTM Standards, 1989) Posing and solving problems should be the major focus of all students' mathematical activity so that through working with interesting, engaging, and intellectually stimulating situations, they come to understand mathematics and use it effectively.

Experiences should be such that students use discovery-oriented, inquiry-based and problem-centered approaches to investigate and understand mathematics. Students should be able to recognize, formulate, clarify and engage in solving problems arising from mathematical situations, everyday experiences, applications to other disciplines, and real-world applications.

MATHEMATICS K–4

I. Problem Solving

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 4, all students should be able to*	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem-solving.
1. A variety of problem-solving strategies (such as making a list, drawing a picture, looking for a pattern, acting out the problem).	a. work individually and with others to use problem- solving approaches to investigate and understand mathematical content (NCTM Standard 1; MO 1.6, 3.5, 3.6, 4.6)	• Given objects and/or pictures that have a variety of attributes such as shapes, colors, and sizes, devise a rule for sorting. Sort the objects and/or pictures using that rule. In spoken or written form explain the rule used (for example, "I put the blue ones together" or "all the big ones belong here.").
2. Computational strategies with whole numbers (addition, subtraction, multiplication and division).	b. use problem solving strategies to construct meaning from mathematical tasks (NCTM Standard 1; MO 1.6, 3.7)	• Sort or classify a set of three-dimensional objects (such as blocks, soup cans, cereal boxes) by common attributes (including sides, ability to roll, and size).
3. When to use concrete objects, calculators, computers, charts, graphs, etc., to organize and solve problems.	c. recognize and define theoretical and actual problems encountered in everyday life, mathematical situations, and various disciplines (NCTM Standard 1; MO 3.1, 3.4)	• Given a menu from a fast-food restaurant, list five ways that you and a friend could eat for $5.00. Compute the cost of each of the five ways.
4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems.	d. develop and apply strategies to predict, prevent, and solve a wide variety of problems (NCTM Standard 1; MO 3.2, 3.3)	
	e. verify, interpret, and evaluate whether a solution addresses the original problem (NCTM Standard 1; MO 2.2, 3.6, 3.7, 3.8)	
	f. select and apply appropriate mathematical tools and technology to solve problems (NCTM Standard 1; MO 2.7)	

MATHEMATICS K–4

I. Problem Solving

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Boxes of the same height are stacked on top of each other in a storeroom. A first set of boxes is 8 inches high, a second set of boxes is 12 inches high, and a third set of boxes is 16 inches high. Design a strategy to determine the height when the tops of all the boxes are even. The height of the storeroom is 10 feet. Is there more than one time when the tops of the boxes would be the same height? Justify your answer. • Given an advertisement from a local toy store, write a story problem that could be solved using both addition and subtraction. Solve the problem. • Put six small counters under one hand, and without looking, move four of them into view. Can you figure out how many are still under your hand? Share your solution with a classmate.

I. Problem Solving

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem-solving.
1. A variety of problem-solving strategies (such as organizing data, drawing a picture, looking for a pattern, writing an expression using a variable).	*By the end of grade 8, all students should be able to* a. use problem-solving approaches to explore and construct meaning from mathematical content (NCTM Standard 1; MO 1.6, 3.4, 3.5, 3.7)	• Identify a fast-food restaurant that offers a "Value Meal" combo. Investigate the restaurant's prices to determine whether the "Value Meal" or a $1.00 off coupon (purchasing the same items) is a better deal. Determine your city's sales tax rate, how the tax is added to the dollar total, and decide if this additional cost affects your solution.
2. Computational strategies with whole numbers, decimals, fractions, and integers.	b. pose authentic problems within and outside the field of mathematics (NCTM Standard 1; MO 2.1, 3.1, 3.4, 3.6)	• Consider the days of the month as "even" days and "odd" days (for example, Jan. 1 is "odd," Jan. 2 is "even"). If you received $1.00 on "odd" days and 50 cents on "even" days, on which day would you have accumulated $50? Explain your process. List the patterns you see in finding the $50 solution. Describe a general rule to determine the amount of money accumulated given a date.
3. Models, calculators, computers, charts, and graphs may be used to organize and solve problems.	c. develop and apply strategies to solve problems, with emphasis on multistep and non-routine problems (NCTM Standard 1; MO 1.8, 3.2, 3.3)	
4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems.	d. analyze, evaluate, and verify results with respect to the original problem (NCTM Standard 1; MO 2.2, 3.7, 3.8)	• Given a 30 cm by 30 cm grid sheet, find the largest volume possible if you were to make an open-ended box (five sides with the top open). Make a chart showing the results of your investigations. Once the largest volume is found, be prepared to justify the results. Change the original grid sheet size, such as, 20 by 20, 20 by 30, and look for patterns in the results. Determine a general rule to describe the results.
	e. transfer strategies to similar problems (NCTM Standard 1; MO 3.2, 3.3, 3.6)	

I. Problem Solving

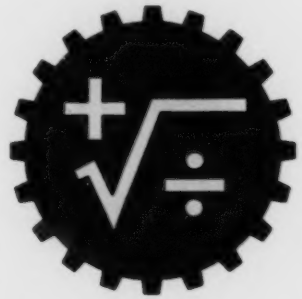
What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Create a scale model of your mathematics classroom. Determine the total area of the room that can be painted. Explain how you would determine the amount of wall space that is to be painted. Investigate the amount of paint necessary to appropriately cover an interior wall. Calculate the number of cans of paint necessary to cover the walls in your mathematics classroom. Estimate the amount of paint necessary to paint another classroom in your school. Can you use your original calculations to estimate the amount of paint needed to paint the interior of your entire school? Explain your reasoning. • A national magazine, *"Middle Schoolers R Us,"* has reported that an 8th grader's potential walking speed is determined by his/her height. Develop a method to test this claim. Conduct the appropriate experiments, then prepare a report of your results. The report should include charts and graphs in addition to the calculations justifying the conclusions. Assuming that speed is a function of height and this is a linear relationship, consider the following: (1) A new student (e.g., Slim Pickens, height 6'5") enrolls in your school. Determine his speed potential. (2) Explain whether a speed of 0 is realistic. (3) Investigate the real-world realistic values for height (domain) and the real-world realistic values for speed (range). Then justify your conclusions. • Plan a home-cooked meal and a similar meal from a fast-food restaurant. Determine which meal is healthier based on nutritional value (percent of fat, percent of salt, etc.). Determine the cost of each meal. Justify whether you should eat out or eat the home-cooked meal.

I. Problem Solving

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem-solving.
By the end of grade 12, all students should know	*By the end of grade 12, all students should be able to*	
1. Problem-solving strategies such as organizing data, drawing a picture, looking for a pattern, modeling, researching, and algebraic strategies.	a. use problem-solving strategies to investigate and understand mathematical content (NCTM Standard 1; MO 1.6, 3.5	• Develop a way to enhance the marketability of a concert, including the number, cost, and types of tickets sold.
2. Computational strategies for the set of real numbers.	b. recognize and formulate problems from situations within and outside mathematics (NCTM Standard 1; MO 3.1, 3.5)	• Develop a brochure and a cost analysis to be used by a sales force for selling tour packages with options.
3. Models, calculators, computers, charts, graphs, etc., may be used as problem-solving tools.	c. organize, develop and apply integrated mathematical problem-solving strategies to solve problems within and outside mathematics (NCTM Standard 1; MO 3.2, 3.3)	• Create a game for a Mathematics Carnival. Investigate and justify the mathematical principles involved in the game. • Design a procedure to examine the cost-effectiveness of games for a Mathematics Carnival.
4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems.	d. apply the process of mathematical modeling to real-world problem situations (NCTM Standard 1; MO 2.1, 3.6)	• Compare and contrast the mathematics and methods used for solving similar problems from different eras.
	e. analyze, evaluate, and reflect upon the process(es) used in solving problems (NCTM Standard 1; MO 2.2, 3.4, 3.6, 3.7, 3.8)	• Given a limited budget, calculate the cost of all materials needed to build a garage, an addition to a room, or a renovation. Create an itemized list of the amount and cost of the materials needed for the project. Specify and justify the dimensions of the garage or the added room or the renovation. • Estimate the amount of concrete needed for a sidewalk 10 feet long. Describe how you determined your estimate.

MATHEMATICS 9–12

II. Communication



K-12 Content Overview

"Mathematics can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematics productively. Communication plays an important role in helping students construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics; it also plays a key role in helping students make critical connections among physical, pictorial, graphic, symbolic, verbal and mental representations of mathematical ideas. When students see that one representation, such as an equation, can describe many situations, they begin to understand the power of mathematics. When they realize that some ways of representing a problem are more helpful than others, they begin to understand the flexibility and usefulness of mathematics." (NCTM Standards, 1989)

Classroom experiences should allow students to model situations and discuss mathematical ideas using oral, written, concrete, pictorial, graphical, and algebraic methods, and a variety of technologies, such as computers, calculators, video, CD-ROM, and videodisks to represent and communicate their mathematical ideas.

II. Communication

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 4, all students should be able to*	NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than communication.
1. The language of mathematics may be used in reading, writing, listening and speaking.	a. relate physical materials, pictures, and diagrams to mathematical ideas (NCTM Standard 2; MO 2.1)	• Use manipulatives (such as, color tiles, or Unifix cubes) to demonstrate the meaning of addition, subtraction, multiplication, and division.
2. Mathematical ideas may be represented by visual models.	b. organize information into useful forms, such as verbal, symbolic, or graphic (NCTM Standard 2; MO 1.8)	• Write, tell, or illustrate a story using mathematical language to describe real-world situations.
3. Mathematical symbols represent real-world situations.	c. reflect on and clarify thinking about mathematical ideas and situations (NCTM Standard 2; MO 2.2)	• Investigate the number of rectangles that can be made using a set number of color tiles. Communicate your results.
4. Information may be organized in a variety of ways.	d. communicate the relationship between everyday language, mathematical language and symbols (NCTM Standard 2; MO 2.3)	• Design a display for, temperature, rainfall amounts, and/or growth of a plant.
		• Read a story or listen to a song and illustrate or discuss the mathematical concepts discovered there.
	e. demonstrate the ability to select and apply appropriate strategies, such as representing, discussing, reading, writing, listening, and using technology in mathematics (NCTM Standard 2; MO 2.2)	• Save the seeds from a piece of watermelon. Count the number of seeds from your watermelon and add your information to a class graph. Read the graph to see whose watermelon had the most seeds and whose had the least seeds.

II. Communication

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than communication.
1. The language of mathematics may be used through reading, writing, listening, and speaking.	*By the end of grade 8, all students should be able to*	• Make a diagram to represent the following situation: There are 30 students in an algebra class. Eleven of these students are in band and 15 play basketball. Five students are in both band and basketball. Write a narrative to explain your diagram.
2. How to represent mathematical ideas with visual models.	a. model situations using oral, written, concrete, pictorial, graphical, technological, and algebraic methods (NCTM Standard 2; MO 1.8, 2.1)	• Design a spinner in two colors, blue and green. Make the spinner in such a way that it is twice as likely to land on blue as on green. Explain your rationale for the way you designed your spinner.
3. Mathematical symbols may be used to represent a variety of situations.	b. reflect on and model mathematical ideas and mathematical situations common to the classroom and the workplace (NCTM Standard 2; MO 2.6, 4.8)	• Given a dart board with the point values of only 4 and 7, and an unlimited number of darts, find the largest impossible score. Describe in writing why the given solution must be the largest impossible score.
4. That information may be organized in a variety of ways.	c. reflect on and clarify their own thinking about mathematical ideas and situations (NCTM Standard 2; MO 2.2)	• Use the information below to construct a mathematical problem and write a narrative of how to solve your problem.
	d. develop common understanding of mathematical ideas, including the role of definitions (NCTM Standard 2; MO 2.2, 2.3)	
	e. draw mathematical ideas and conclusions from reading, listening, and viewing (NCTM Standard 2; MO 3.5, 4.1)	
	f. discuss mathematical ideas, make conjectures, and present convincing rationales (NCTM Standard 2; MO 2.4)	
	g. connect mathematical notation and its role in the development and structure of mathematical ideas (NCTM Standard 2; MO 1.6, 1.9, 2.4)	

Ice cream	Cost per serving	Flavor rating
Cherry Berry	$1.25	7
Chocolate Rave	.89	4
White Sparkle	.95	3
Candy Cane	1.09	3
Fruit 'n' Fun	.85	7

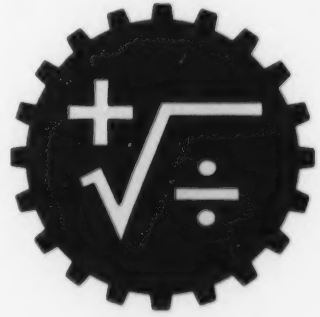
II. Communication

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than communication.
1. The language of mathematics may be used through reading, writing, listening, and speaking.	*By the end of grade 12, all students should be able to*	• Read a piece of literature such as *Gulliver's Travels* or *Through the Looking Glass* looking for relationships in mathematics. Discuss how mathematics is used in the literature. Write an essay or short story describing situations, visual images and objects in terms from mathematics using both numerical relationships and geometric relationships.
2. Mathematical ideas may be represented with visual models.	a. reflect upon and clarify thinking about mathematical ideas and relationships (NCTM Standard 2; MO 1.6, 2.2)	• Design a three-dimensional scale model to illustrate a given structure such as a shopping mall, an amusement park, or a sports arena.
3. Mathematical symbols may be used to represent a variety of situations.	b. interpret generalizations discovered through investigations to formulate, revise, and adjust mathematical definitions (NCTM Standard 2; MO 1.2, 1.7, 2.2)	• Design an acceptable popcorn container to hold the greatest amount of popcorn and use the least possible amount of materials.
4. Information may be organized in a variety of ways.	c. visualize mathematical ideas by reading about, listening to, or viewing concrete models (NCTM Standard 2; MO 1.9, 2.4)	• Contact a business such as a financial institution, construction company, local industry, or chamber of commerce to collect information so you can prepare charts and graphs representing information collected for a presentation or to convince a customer to buy your product.
	d. plan and create effective verbal and non-verbal forms of communicating mathematics for a variety of purposes and audiences (NCTM Standard 2; MO 2.1)	• Using number sets such as natural, whole integer, rational, irrational, and real numbers, design some diagram, model or video to describe the relationships of the sets.
	e. present mathematical ideas and logical justifications, both written and oral (NCTM Standard 2; MO 2.1, 3.5, 4.1)	
	f. ask clarifying and extending questions about the mathematics read about, heard about, or viewed through models (NCTM Standard 2; MO 2.3)	

II. Communication

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	g. recognize the economy, power, and elegance of mathematics notation and its role in the development of mathematical ideas (NCTM Standard 2; MO 1.6, 1.9, 2.4) h. read, write, and talk about mathematical ideas as they relate to real-life applications and multiple workplace situations (NCTM Standard 2; MO 1.10, 2.6, 3.2, 4.8)	

III. Reasoning



K-12 Content Overview

Every student should have the opportunity to develop higher order thinking skills. The key is to unlock the world of mathematics through a student's natural inclination to strive for purpose and meaning. Reasoning is fundamental to the knowing and doing of mathematics. Demonstrating the logical validity of conjectures is the essence of the creative act of doing mathematics. To give more students access to mathematics as a powerful way of making sense of the world, it is essential that an emphasis on reasoning pervade all mathematical activity. Also important is the goal that all students become confident, self-reliant mathematical thinkers. Students need to develop the capability to confront a mathematical problem, persevere in its solution, and evaluate and justify their results.

Students should learn to appreciate the pervasive use and power of reasoning as part of mathematics, develop the habit of monitoring and validating their own thinking, and use their mathematical reasoning skills in other disciplines in their lives.

MATHEMATICS K-12

III. Reasoning

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know 1. Objects/numbers may be used in more than one way to determine or construct relationships between and among them. 2. Results must be verified. 3. Data may be organized in a variety of forms to look for patterns. 4. Geometric and number properties.	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 4, all students should be able to* a. draw logical conclusions about mathematics (NCTM Standard 3; MO 3.5) b. use models, known facts, properties, and relationships to explain their thinking (NCTM Standard 3; MO 4.1) c. justify answers and solution processes in an organized and convincing way (NCTM Standard 3; MO 1.8, 3.4, 3.7, 4.1) d. use patterns and relationships to analyze mathematical situations (NCTM Standard 3; MO 1.6)	NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than reasoning. • Given the following statements, find the secret number: (a) The number is odd. (b) The number is less than 40 and greater than 30. (c) The number is not 33. (d) The sum of the digits is between 5 and 10. • Prepare a set of three clues similar to the ones presented in activity #1 that will describe a secret number. • What could be the next number in the sequence, "2, 4,..."? Explain your answer. Can you justify any other numbers? Explain all possibilities you find. • Determine how much food a given pet would eat in a year and estimate the total cost. Explain how you reached your answer. • Play card games such as "War" or "Double War." Tell how you decided who won the game.

III. Reasoning

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Play strategy games such as tic-tac-toe, NIM games, checkers, chess or other games requiring the use of or development of winning strategies. Explain your winning strategies for the game you selected. • Play "Guess My Rule" with a set of blocks or objects. One person thinks of an attribute (color, shape or size). The others determine the chosen attribute by placing the blocks within a designated area. Objects are placed in or out of the area until enough information is acquired to guess the attribute.

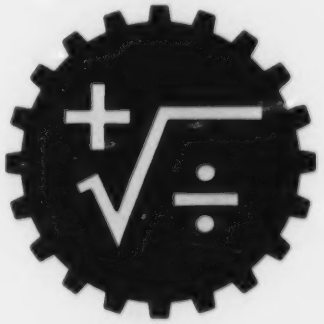
III. Reasoning

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 8, all students should be able to*	NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than reasoning.
1. Information may be organized in a variety of forms to look for patterns and relationships.	a. make and interpret mathematical conjectures and rationales for a conclusion (NCTM Standard 3; MO 1.5)	• Collect data on the number of windows on one floor or part of your school and estimate how many windows are in the building. Justify your reasoning.
2. Results must be justified.	b. justify their own thinking (NCTM Standard 3; MO 4.1, 4.4)	• Look at patterns of numbers and determine possible next terms in the pattern. Predict the general formula that fits the pattern.
3. Geometric and number properties.	c. use reasoning processes in regard to spatial reasoning and reasoning with proportions and graphs (NCTM Standard 3; MO 1.6, 1.8, 3.5)	• Make a scale drawing of a roller coaster and sketch a graph to represent the speed of a roller coaster versus its position on the track. Explain the reasoning used to construct your graph. Interpret the graph.
	d. recognize and apply deductive and inductive reasoning (NCTM Standard 3; MO 3.5, 4.1)	• Use or generate data to predict some future result. Continue the experiment to test conjectures. Analyze the results by comparing conjectures to results and describe how (and if) a revision is necessary to improve future predictions.
	e. use patterns and relationships to generalize an algebraic representation (NCTM Standard 3; MO 1.6)	

III. Reasoning

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know 1. Information may be organized in a variety of forms to look for patterns and relationships. 2. Results must be justified. 3. Geometric and number properties. 4. The process of inductive reasoning. 5. The process of deductive reasoning.	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 12, all students should be able to* a. make and test conjectures (NCTM Standard 3; MO 1.7) b. defend the validity of their conclusions using mathematical strategies (NCTM Standard 3; MO 3.4, 3.7, 3.8, 4.1) c. follow the mathematical reasoning of others and determine validity (NCTM Standard 3; MO 1.5, 2.3) d. apply inductive and deductive reasoning (NCTM Standard 3; MO 3.5)	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than reasoning. • Prepare an organized and convincing report analyzing the effect of the national debt on the country, state, community and family. • Evaluate and discuss the validity of mathematical reasoning by the authors of news articles; bring in articles for presentations before the class in which the author's mathematical validity, or lack thereof, is analyzed. • Demonstrate how categorizing, drawing schematics, or other organizing systems help to facilitate the reasoning process. Examples include the properties of numbers, categorization of geometric shapes, paths and graph theory.

IV. Connections



K-12 Content Overview

Although it is often necessary to teach specific concepts and procedures, mathematics must be approached as a whole. Concepts, procedures and intellectual processes are interrelated. In a very real sense, "the whole is greater than the sum of its parts." This strand includes a deliberate attempt, through specific learning activities, to connect ideas and procedures both among different mathematical topics and with other content areas.

Instructional strategies should allow students to view mathematics as an integrated whole; link conceptual and procedural knowledge; understand and use various representations of concepts and connect them to one another; explore problems and describe and confirm results using various representations; use models, calculators, and other technologies to connect different representations of mathematical concepts; and use one mathematical idea to extend understanding of another, related mathematical idea.

Students should be able to connect mathematics to the real world by recognizing the connection between mathematics and other disciplines and applying mathematical thinking and problem-solving in those areas. They should also recognize the role of mathematics in their daily lives, in careers, and in society and apply mathematics in those contexts.

IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than connections.
1. Problems may be looked at in more than one way.	*By the end of grade 4, all students should be able to*	• Read literature such as *Anno's Counting House, Ten Black Dots, The Button Box, 17 Kings 42 Elephants, How Big is a Foot, The Door Bell Rang*. (Many activities involving mathematics will arise during the reading of such books.)
2. Mathematics is used in other subject areas.	a. link concepts to student-generated procedures (NCTM Standard 4; MO 1.6, 1.10, 2.2)	• Given a floor plan of your school, find the shortest walking path from your classroom to the cafeteria. Then find the shortest path to the cafeteria that goes by a rest room.
3. Mathematics is used in the real world.	b. relate various representations of concepts or procedures to one another using a variety of methods, forms, and technologies (NCTM Standard 4; MO 1.6, 2.7)	• Your school's parent group has donated funds to purchase new playground equipment. Design a model of the equipment and the playground.
	c. recognize relationships among different topics in mathematics (NCTM Standard 4; MO 1.6, 1.10)	• Create a time-line for the history of the school.
	d. use mathematics in other curriculum areas and in daily living (NCTM Standard 4; MO 1.10, 4.7)	• Collect or record data gathered in other disciplines.
		• Explore numeric and geometric patterns.

IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• After attending a career/hobby day, discuss how mathematics is used in the careers and hobbies you investigated. • Identify real-world situations in which mathematics is used.

IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know 1. Problems may be looked at in more than one way. 2. Mathematics is used in other subject areas. 3. Mathematics is used in the real world.	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 8, all students should be able to* a. investigate problems utilizing graphical, numerical, physical, algebraic, and mathematical models or representations to understand and describe conclusions (NCTM Standard 4; MO 1.2, 1.6, 2.1, 3.7) b. apply mathematical reasoning and modeling to solve problems from other disciplines, such as art, music, psychology, science, and business (NCTM Standard 4; MO 1.2, 1.3, 3.2, 3.5) c. use technology such as scientific and graphing calculators, computers and models to demonstrate understanding of mathematical ideas (NCTM Standard 4; MO 1.4, 2.7) d. explore and investigate the importance of mathematics in their lives, future careers, and our ever-changing global society (NCTM Standard 4; MO 4.2, 4.3, 4.7, 4.8)	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than connections. • Interpret a graphical representation or model, such as a pie chart, indicating music preference of students at a local middle school, and portray the information using a different representation or numerical model, physical, or algebraic. • Investigate the mathematics (such as patterns and geometry) involved in a real-life situation, for example, why airlines use "hub" cities in scheduling their routes, or why pictures/frames are manufactured to certain proportions. • Use mathematical models and ratios to investigate the relationship between naturally occurring situations such as wing span vs. body length for several bird species. Given the data generated, predict the wing span for a new species of bird or predict arm span for a person with a specific height. • Investigate, then utilize, the Conservation Department's methods of estimating the number of fish of a certain species in an area lake. Justify your results and discuss how you could verify your predictions.

IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Utilize current technology to investigate how changing the scale on a graph, or using a different representation such as bar graphs, pie graphs, or box graphs, may alter the appearance and impact of the data. • Investigate data from the previous 10-20 years involving local or state citizens, such as quantity of fresh water consumed annually, number of schools/classrooms needed, or number of housing units needed; then predict the amount that will be necessary 10 years in the future, considering population trends.

IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than connections.
1 Problems may be looked at in more than one way.	*By the end of grade 12, all students should be able to*	• Find the Olympic gold medal times for the men's 100 meter dash and display the data in a table and scatter plot. Use a graphics calculator to calculate and display the curve of best fit. Predict the gold medal time for the year 2020.
2. Mathematics is used in other subject areas.	a. recognize and/or derive equivalent representations for a concept (NCTM Standard 4; MO 1.6)	• Investigate the volume of an open-ended box that may be constructed from poster board measuring 20 by 30 centimeters. Discuss the method(s) used to find the dimensions allowing for the maximum volume. Justify your solution.
3. Mathematics is used in the real world.	b. analyze and relate procedures in multiple representations (NCTM Standard 4; MO 1.5, 3.6)	• Use land surveys to calculate areas using two different methods. Explain, contrast, and compare the two methods.
	c. relate and describe the connections within topics of mathematics and other disciplines (NCTM Standard 4; MO 1.6, 1.8, 1.10)	• Investigate recursion by comparing a head of broccoli to its flowerets and the flowerets to its subsets. Write about or discuss the patterns observed. Do further research on recent findings in the area of fractals, chaos and recursion.
	d. investigate and determine the importance of mathematics in their lives, future careers, and our ever-changing global society (NCTM Standard 4; MO 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8)	• Investigate the volume of your local phone book. Estimate the total volume of all phone books to be recycled in your school and community. Research the amount of landfill rescued as a result of this recycling project. Compare and contrast disposal methods to project the cost benefits to your community and the environment.
	e. evaluate the logic and aesthetics of mathematics as they relate to the universe (NCTM Standard 4; MO 1.10, 2.4)	

MATHEMATICS 9-12

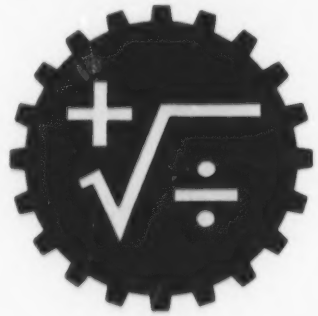
IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Play Monopoly by the rules, but pay for transactions by writing checks (Four to six students per game.) 1. Select a banker. 2. Distribute money by the rules of the game. 3. Players must fill out deposit slips to deposit the money in the bank. 4. Players keep track of transactions on a bank record sheet. The banker also keeps track of all the transactions for all the players in the game. 5. Students may keep some small bills to pay fines. All other transactions must be completed by check. 6. Students reconcile their bank statements with their records. 7. The winner is determined by the rules of the game. The assessment is based on the students' banking records and whether the checks were written correctly. NOTE: Contact a local bank to receive banking materials (check books with checks, deposit slips, and check registers). • List five occupations that never use mathematics and/or computers. List the mathematics actually used in an occupation that seems mathematics-free such as a gift wrapper in a department store. Write a paragraph describing the career you might want to pursue and the use of mathematics and/or computers in that occupation.

IV. Connections

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Investigate, analyze, and predict workplace safety using data from OSHA. • Investigate the impact of a 5¢ national sales tax considering your family, local community, state, and nation. • Use the stock market section of the newspaper to select the stock of a company to follow. Keep track of the stock for several weeks. Make charts and graphs to display the information. Analyze the data to determine the trends for increases and decreases. Justify your analysis.

V. Number Sense
(Show-Me Standards, Math 1)



K-12 Content Overview

Number sense is a quality that successful users of mathematics possess. It is often identified as an intuitive feel for numbers and their variety of interpretations as well as a common sense approach to using numbers. When someone chooses to use fractions in one situation and decimals in another because the respective operations are easier to perform or the results are easier to understand, that process is evidence of good number sense. When students continue to work on a problem until they recognize that their answers are reasonable in the problem's context, they are using good number sense. Intelligent use of the decimal number system and powers of 10 and the ability to use approximations and estimation when appropriate are other indicators of a well-developed number sense.

Computation should support meaningful experiences in geometry, probability, measurement, and other areas of mathematics. Estimation should be used to solve problems for which exact answers are inappropriate and for checking computation results. As students begin to understand the meaning of operations and develop a concrete basis for validating symbolic processes and situations, they should design their own algorithms and discuss, compare and evaluate them. Students should analyze the ways the various algorithms work and how they relate to the meaning of the operation and to the numbers involved.

Number sense is not inherent in a person's ability to perform numerical computations. A "sense-building mode" is best established when students are provided with opportunities to explore number relationships, are encouraged to challenge and question, and are allowed to experiment to discover strategies and techniques of their own that ease the path to the solution of mathematical problems.

Students will only develop strong number sense to the extent that their teachers use pedagogy that encourages the understanding of mathematics as opposed to the memorization of rules and mechanical application of algorithms. Every child has the capability to succeed as a user of mathematics but the degree of success is directly related to the strength of their number sense. The way to ensure that all students acquire a good sense of number is to have them consistently engage in activities that require them to think about numbers and number relationships and to make the connection with quantitative information encountered in their daily lives.

V. Number Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than number sense.
1. Counting and grouping strategies.	*By the end of grade 4, all students should be able to*	
2. Mental computation and estimation strategies.	a. model, explore, develop, and explain number operations for whole numbers (NCTM Standard 7; MO 1.6, 2.1, 3.3)	• Use manipulatives such as color tiles, and unifix cubes to show the relationships among addition, subtraction, multiplication, and division.
3. Place value.	b. use technology to explore numbers (NCTM Standard 6; MO 1.4, 1.6, 2.7)	• Use manipulatives to demonstrate an understanding of place value by modeling operations on numbers.
4. Basic computation facts of addition, subtraction, multiplication, and division with whole numbers.	c. use physical models and real-world experiences to construct number meanings (NCTM Standard 5; MO 1.10, 2.3, 4.1)	• Use manipulatives or models to demonstrate fractional parts and the fractional notations representing 1/2, 1/3, 2/4, etc. Show fractional parts that are equivalent. Explain how you know they are equivalent.
5. Addition and subtraction of fractions with like denominators.	d. demonstrate an understanding of our numeration system by relating counting, grouping, and place value concepts (NCTM Standard 6; MO 1.6, 3.6, 4.1)	• Skip count using a calculator. Record the data. Examine the units place for patterns. Describe the patterns.
6. U.S. customary and metric units of measure.	e. utilize number sense to develop number meanings and explore number relationships (NCTM Standard 6; MO 1.6, 3.3)	• Using a Frisbee or paper plate, estimate how far it can be thrown. Throw and give a better estimate. Now measure the actual distance. Repeat the experiment several times and average the data.
7. The appropriate use of calculators.		

V. Number Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	f. use a variety of mental computation and estimation strategies to solve specific problems (NCTM Standard 5; MO 1.10, 3.3, 4.1) g. demonstrate an understanding of the attributes of length, capacity, weight, area, volume, time, temperature, and angle (NCTM Standard 5; MO 1.6, 4.1) h. make and use standard and nonstandard measurements in problems and everyday situations (NCTM Standard 5; MO 3.2, 3.3) i. explore the concepts of fractions, mixed numbers, and decimals and be able to apply them to problem situations (NCTM Standard 12; MO 1.6, 3.2, 3.3, 4.1)	• Estimate the number of raisins in a 1/2 oz. box. Record estimates. Open the box and examine the raisins in view. (Do not empty the box.) Revise estimates if necessary. Count raisins. (A third estimate can be made when raisins are dumped from the box.) • Create a chart to help the shipping clerks of Yummy Gum speed up their work. Yummy Gum ships gum by packs and single pieces. Each pack contains 10 pieces of gum. Create a chart or a fast way that the shipping clerks can package and send off the orders. • With a partner, a bowl with 20 cubes and a die, guess the number of rolls it will take to empty the bowl. Roll the die and remove that number of cubes from the bowl. After rolling the die the number of times you guessed, record "yes" or "no" to indicate if the bowl was emptied. (from *About Teaching Mathematics*, 1992)

V. Number Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than number sense.
By the end of grade 8, all students should know	*By the end of grade 8, all students should be able to*	
1. Addition, subtraction, multiplication, and division with rational numbers.	a. extend understanding and ability to apply whole number operations to all rational numbers (such as integers, and fractions, decimals) (NCTM Standard 6; MO 3.2, 3.3, 4.1)	• Provided with retail lists, including prices and the sales tax rate, estimate the total cost of selected items. Explain your strategy.
2. Numbers and their relationships can be represented in multiple forms.	b. use multiple representations of equivalent forms of numbers such as integers, fractions, decimals, percent, exponents, and scientific notation in a variety of situations (NCTM Standard 5; MO 1.6, 1.10)	• Explore the local forms of media to find examples of how fractions, decimals, and percents are used. Justify the appropriateness of the media's use of the numbers.
3. The appropriate use of technology.		• Investigate the relationship between perimeter and area of closed figures and generalize the results.
	c. describe connections and relationships of numbers such as ratios, proportions, and percents in problem situations (NCTM Standard 5; MO 1.6, 2.1)	• Using ratios, proportions, or a method of choice, predict a future situation, such as population of Missouri in the year 2010, the number of bald eagles nesting in Missouri in 2000, on the year someone will run the mile in 3 minutes flat.
	d. investigate number forms such as fractions, decimals, and percents, and demonstrate their use in today's society (NCTM Standard 5; MO 1.10, 2.2, 4.1)	• Aiming for a target interval, enter a number into a calculator. Use only the operation of multiplication to arrive at a number within the interval. Compare the impact of multiplying by a number less than one and a number greater than one.
		• Select a rational number. Show three other representations of the same number.

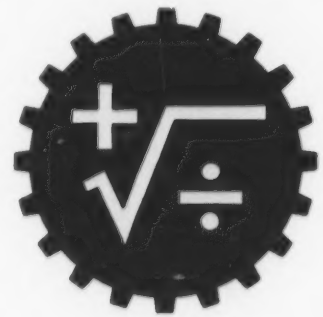
V. Number Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	e. develop, analyze, and explain procedures for computation and techniques for estimation (NCTM Standard 7; MO 2.3, 3.3) f. develop, analyze, and explain methods for solving proportions (NCTM Standard 7; MO 2.3, 3.3) g. check and explain the reasonableness of solutions, strategies, and results (NCTM Standard 7; MO 1.7, 3.3, 3.7, 4.1) h. select appropriate methods of computation, such as mental arithmetic, estimation, calculator, computer, and paper/pencil to reflect upon solutions, strategies and results (NCTM Standard 7; MO 3.3, 3.6, 3.7) i. represent numerical relationships in one- and two-dimensional graphs (NCTM Standard 5; MO 2.1, 2.7)	• Play Monopoly by the rules, but pay for transactions by writing checks (four to six students per game). 1. Select a banker. 2. Distribute money by the rules of the game. 3. Players must fill out deposit slips to deposit the money in the bank. 4. Players keep track of transactions on a bank record sheet. The banker also keeps track of all the transactions for all the players in the game. 5. Students may keep some small bills to pay fines. All other transactions must be completed by check. 6. Students reconcile their bank statements with their records. 7. The winner is determined by the rules of the game. The assessment is based on the students' banking records and whether the checks were written correctly. NOTE: Contact a local bank to receive banking materials (check books with checks, deposit slips, and check registers).

V. Number Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than number sense.
1. Addition, subtraction, multiplication, and division with real numbers.	*By the end of grade 12, all students should be able to*	
2. Numbers and their relationships can be represented in multiple forms.	a. develop, analyze, and explain procedures used for representing and analyzing relationships in tables, verbal rules, equations, and graphs (NCTM Standards 5 and 6; MO 1.6, 1.8, 1.10, 2.6)	• Research and discuss the relationship of various currency systems and difficulties encountered in the exchange between systems.
3. The appropriate use of technology.	b. analyze the effects of parameter changes on the graphs of functions (NCTM Standards 5 and 6; MO 1.6, 3.6, 4.1, 4.7)	• Examine the effects of changing parameters in compound interest formulas, growth patterns/half-life formulas, fractals and chaos equations. Describe how the changes affect a specific situation.
4. Data can be organized in many forms.	c. analyze and describe relationships and the resulting effects between changes in an independent variable and a dependent variable (NCTM Standards 5 and 6; MO 1.6, 3.3, 4.1)	• Given a problem situation involving complex computations, determine a range within which a reasonable answer could be expected to fall. Explain why you selected the range.
		• Discuss whether the method used to present mathematical data influences numerical sense of understanding (such as, different scales on the axes of a graph or changing window on a graphing calculator).
		• Discuss the significance of a small numerical error compounded over a long distance or a long time (such as, a small angular error when traveling to the moon in a space ship).

VI. Geometric and Spatial Sense
Show-Me Standards, Math 2



K-12 Content Overview

Geometry is the systematic study of spatial relationships. It is connected to every strand in a mathematics curriculum and to a vast multitude of situations in real life. Well-constructed diagrams allow us to apply knowledge of geometry and geometric reasoning and intuition to arithmetic and algebra problems. Whether designing an electronic circuit board, a building, a dress, an airport, a bookshelf or a newspaper page, a solid understanding of basic geometric principles is required.

Traditionally, geometry in schools has been taught as the prime example of a formal deductive system. While this view of the content is important, its domination has led to the exclusion of other, less readily formalized topics and applications. Geometry instruction should not be limited to formal deductive proof and simple measurement activities, but should include the study of geometric transformations, analytic geometry, topology, and the connection of geometry with algebra and other areas of mathematics.

By virtue of living in a three-dimensional world, having dealt with space for 5 years, children enter school with a remarkable amount of intuitive geometric knowledge. The geometry curriculum should take advantage of this intuition. In early elementary school, a rich, qualitative, hands-on study of geometric objects helps young children develop spatial sense and a strong intuitive grasp of geometric properties and relationships. Eventually they develop a comfortable vocabulary of appropriate geometric terminology. In the middle school years, students should begin to use their knowledge in a more analytical manner to solve problems, make conjectures, and look for patterns and generalizations. Gradually they develop the ability to make inferences and logical deductions based on geometric relationships. In high school, the study of geometry expands to include coordinate geometry, trigonometry, and both inductive and deductive reasoning.

MATHEMATICS K-12

VI. Geometric and Spatial Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than geometric and spatial sense.
By the end of grade 4, all students should know	*By the end of grade 4, all students should be able to*	
1. Standard and nonstandard units of measure.	a. describe, model, draw, and classify shapes (NCTM Standard 9; MO 1.4, 1.6, 2.1)	• A dog is tied to a 5-meter rope at the middle of the 10-meter-long side of a garage. Make a sketch of the outer path on which the dog can walk. Move the tie post to the corner of the garage. Compare and contrast the outer path of the two situations.
2. Descriptions of two- and three-dimensional figures.	b. investigate and predict the results of combining, subdividing, and changing shapes (NCTM Standard 9; MO 1.1, 1.6, 3.1)	• Classify shapes by matching concrete objects, such as tangrams, pattern blocks, or cutouts to their name. Create a design using one of each shape.
3. Geometric shapes are found in the real world.	c. visualize, draw, and compare shapes (NCTM Standard 9; MO 1.8, 2.1, 3.2, 3.3)	
4. Objects can be located by relative position.	d. connect geometric ideas to number and measurement ideas (NCTM Standard 9; MO 1.6, 3.5, 4.1)	• Given a model, use interlocking cubes to build a three-dimensional replica of the model.
5. The process of measurement.	e. explore geometry in their world (NCTM Standard 9; MO 1.10, 2.4)	• Use a nonstandard unit of measure (such as beans) to find the area of a leaf.
	f. investigate concepts of lines, angles, similarity, congruence, and symmetry (NCTM Standard 9; MO 1.6, 2.5)	• Use a nonstandard unit of measure (such as beans, hands, feet, paper clips) to find a classmate's height; the height of your desk; the length and width of your desk; the area of a foot print, a hand or a leaf; the mass or weight of classroom objects (using a balance scale).
	g. investigate length, capacity, weight, mass, area, volume, time, and temperature (NCTM Standard 10; MO 1.6, 2.5)	

VI. Geometric and Spatial Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	h. use standard and nonstandard units of measure (NCTM Standard 10; MO 1.10) i. locate objects by relative position including top, bottom, left, right, over and under (NCTM Standard 9; MO 1.6)	• Explore the letters of the alphabet or geometric shapes (such as the shapes contained in a bucket of pattern blocks) for mirror symmetry. Sort the letters into categories that have one line of mirror symmetry, more than one line of mirror symmetry, and those letters that do not have mirror symmetry. • Build a shape (two-dimensional) or structure (three-dimensional) from a description provided by another student. Build a shape or structure with a set of materials that cannot be seen by the other student. Another student with an identical set of materials tries to build the shape or structure with only verbal directions. (For younger children, teacher gives description.) • Copy a partner's design onto another geoboard or dot paper. • Make a shape (perhaps something that can fly) on your geoboard. Describe how shapes are alike and how they are different. Then sort and classify the shapes. • Play "I'm Thinking of an Object." Give clues about the location of the object you are thinking about. The classmate who guesses what you are thinking about then takes a turn.

VI. Geometric and Spatial Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than geometric and spatial sense.
1. Structures of measurement systems.	*By the end of grade 8, all students should be able to*	
2. Descriptions of two- and three-dimensional shapes and their relationships .	a. identify, describe, compare, classify, and represent geometric figures (NCTM Standard 12; MO 1.4, 1.6, 2.1)	• Use straws to construct polyhedra and classify according to their attributes.
3. Geometric shapes are found in the real world.	b. explore transformations of geometric figures (NCTM Standard 12; MO 1.6)	• Use modeling or simulation to compare the sum of the length of two sides of a triangle to the third side. Justify your results.
	c. investigate and apply geometric properties and relationships (NCTM Standard 12; MO 1.6, 2.4, 3.6)	• Draw a single figure on graph paper. Use reflections, slides and turns to make a wall paper design.
	d. use geometry to describe their world (NCTM Standard 12; MO 1.10, 2.4)	• Collect examples of geometry used in architecture. Describe geometric characteristics such as the Golden Rectangle.
	e. extend their understanding of the process and structure for measurement (NCTM Standard 13; MO 1.4, 2.6, 2.7)	• Given 90 yards of fencing, design a fence to enclose a swimming pool to allow for the following: the largest possible area, the most visually pleasing shape, and the most creative shape. Justify how your shape fits each category.
	f. select and discuss appropriate units and devices to estimate or make measurements, considering degree of accuracy (NCTM Standard 13; MO 2.6, 3.1, 3.7, 4.1)	

VI. Geometric and Spatial Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	g. apply the concepts of perimeter, area, volume, angle measure, capacity, weight, and mass (NCTM Standard 13; MO 2.5, 3.8, 4.1) h. investigate the concept of rate of change (NCTM Standard 13; MO 1.4, 1.6, 1.8) i. develop formulas and procedures for determining measures to solve problems (NCTM Standard 13; MO 1.4, 1.6, 1.8, 3.7)	• Given a commercial cereal box, calculate the potential volume. Investigate how changing the dimensions of the box will affect the volume. Using the given amount of pasteboard, determine the largest possible volume. Discuss why dimensions of a current box are used instead of another dimension.

VI. Geometric and Spatial Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know 1. Structures of geometric and measurement systems. 2. Properties and relationships of two- and three-dimensional shapes. 3. Geometric shapes can be used to describe the real world.	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 12, all students should be able to* a. interpret and draw three-dimensional objects (NCTM Standard 7; MO 1.5, 1.9, 2.7) b. represent and solve problem situations with geometric models and apply properties of figures (NCTM Standard 7; MO 1.5, 2.7, 3.7) c. classify figures in terms of congruence and similarity and apply these relationships (NCTM Standard 7; MO 1.1, 1.4, 1.6, 3.5) d. deduce properties of, and relationships between, figures from given assumptions (NCTM Standard 7; MO 1.6, 1.8, 2.4, 3.5) e. translate between synthetic and coordinate representations using a variety of methods and technologies (NCTM Standard 8; MO 1.4, 2.7) f. deduce properties of figures using transformations and coordinates (NCTM Standard 8; MO 2.4, 3.5)	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than geometric and spatial sense. • Build models of the five regular polyhedra. Discuss the relationships between the number of faces, edges, and vertices. Through exploration, pose the question as to why there are only five regular polyhedra. • Build three-dimensional objects following a model, then draw the front, right, top, and base views to demonstrate skills in spatial visualization with a partner. Each two-person team will then create a base view for another team to build. • Determine the smallest amount of material needed to produce a can with a given volume Justify your results. • Use paper-folding activities and/or computer technology to deduce properties and relationships between figures (such as exploring the relationships of opposite sides, opposite angles, diagonals of the quadrilateral family, relationships between angles, chords of a circle, etc.). Develop a simple deductive system using these relationships. • Create a design to tile a floor or a wallpaper pattern using tessellations.

VI. Geometric and Spatial Sense

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	g. identify congruent and similar figures using transformations (NCTM Standard 8; MO 1.5, 3.5, 3.6)	• Design a packing box that can be used to ship eight small spheres (such as decorative ornaments) using the least amount of material. Justify your results.
	h. analyze properties of transformations and relate translations to vectors (NCTM Standard 8; MO 1.6, 2.4, 3.6, 4.1)	• Determine the angle of depression between two markers with different elevations on a contour map.
	i. apply an understanding of perimeter, area, volume, angle measure, capacity, weight and mass (NCTM Standard 7; MO 2.5, 3.3, 4.1)	• Determine the area of an irregularly shaped region (such as plans from a registered land surveyor).
	j. model, describe, and analyze maximum and minimum points on a graph (NCTM Standard 13; MO 1.6, 2.1, 4.3, 4.7)	• Find the limit of the area of a circle by constructing an inscribed and/or a circumscribed convex polygon and determining its area. Repeat the process increasing the number of sides of the polygon. Explain why the limit is numerically sensible.
	k. model, describe, and analyze patterns of sequences through processes of geometric change, approximations, and limits (NCTM Standard 14; MO 1.6, 2.1, 4.3, 4.7)	• Draw a triangle on a sheet of graph paper. Draw images using a rotation, a reflection, and a translation.
	l. recognize and apply trigonometry to problem situations (NCTM Standard 9; MO 3.1, 3.6, 4.2, 4.8)	

VII. Data Analysis, Probability and Statistics
Show-Me Standards, Math 3

K-12 Content Overview

Probability is the study of random events. It is used in analyzing games of chance, genetics, weather prediction, and a myriad of other everyday events. Statistics is the mathematics we use to collect, organize and interpret numerical data. It is used to describe and to analyze sets of test scores, election results and shoppers' preferences for particular products. Probability and statistics are closely linked because statistical data are frequently analyzed to see whether some conclusions can legitimately be drawn about a particular phenomenon and also to make predictions about future events.

Understanding probability and statistics is essential in the modern world with the print and electronic media full of statistical information and its interpretation. The goal of mathematical instruction in this area should be to make students sensible, critical users of probability and statistics, able to apply their processes and principles to real-world problems. Students should be able to judge whether the statistics are meaningful and used appropriately in an argument in order to make a decision on that argument.

Elementary children are able to understand the probability of an event. Experiments leading to discussions about the difference between experimental and theoretical probability can be done by older elementary and middle grade students. Young children can start out as early as kindergarten with data collection, organization, and graphing and a focus on those skills, with obviously increasing sophistication, should last throughout their schooling. Whenever students look at data, there should be some question they are trying to answer or some position they are trying to support.

Probability and statistics hold the key for enabling our students to understand, process and interpret the vast amounts of quantitative data that exist all around them. To be able to judge the truth of a data-supported argument presented to them, to discern the believability of a persuasive advertisement that talks about the results of a survey of all of the users of a particular product, or to be knowledgeable consumers of the data-intensive government and electoral statistics that are ever-present, students need the skills that they can learn in a well-conceived probability and statistics curriculum strand.

VII. Data Analysis, Probability and Statistics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than data analysis/ probability and statistics.
1. Strategies to collect data.	*By the end of grade 4, all students should be able to*	
2. Strategies to organize data.	a. collect, organize, and describe data through the use of technologies and other resources (NCTM Standard 11; MO 1.1, 1.3, 1.4, 1.8)	• Collect data on a given topic and classify the data based on similarities.
3. Different ways of displaying data.	b. construct, read, and interpret displays of data through verbal, nonverbal, symbolic, and graphic forms (NCTM Standard 11; MO 1.5, 3.3, 3.6, 4.1)	• Use collected data to construct a variety of graphs or charts. Discuss which representation best displays the information.
4. The appropriate display of data.	c. solve problems that require collecting and analyzing data (NCTM Standard 11; MO 2.3, 3.2, 3.3, 4.3)	• Make a prediction on an upcoming event based on data collected and analyzed from a past series of events.
5. The appropriate use of technology.	d. explore concepts of chance (NCTM Standard 11; MO 1.6, 1.7, 4.3, 4.7)	• Given a spinner that is divided into four equal sections with two blue sections, one red section, and one green section, how many times would you expect to get green if you spin 20 times? Explain your answer. Now spin 20 times and record your results. Compare the results to your prediction.
		• Make a spinner with four equal parts. Spin the spinner 20 times and record the number of times each area of the spinner comes up. Repeat the above with four unequal parts on a spinner. Record your data.

VII. Data Analysis, Probability and Statistics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Make a graph of the colors of socks students in your class wear to school on a given day. Explain the graph to a partner. • Create an attendance graph to collect data for the class lunch count. Interpret the graph to help your teacher complete the lunch count report. • Collect and interpret information from an opinion graph (such as favorite color, cartoon, movie, place to eat, etc.). • Each pair of students needs 5 pennies and 5 dimes, a recording sheet for each and a piece of 8 1/2 x 11 paper (target). With your partner, toss all 10 coins onto the target paper. Then record in the appropriate column on the recording sheet the number of dimes and pennies that landed on the target. • When the sheet is filled, add up the amount of money that hit the target for each player to find out who is the winner.

MATHEMATICS K-4

VII. Data Analysis, Probability and Statistics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than data analysis/ probability and statistics.
1. Standard measures of central tendency.	*By the end of grade 8, all students should be able to*	
2. Methods to analyze data.	a. develop, analyze, and explain methods utilized to collect, organize, and describe data (NCTM Standard 10; MO 1.1, 1.4, 1.8, 2.1)	• Organize student-generated data from investigations of real-world situations (such as height vs. shoe size, predicted vs. actual colors in a bag, or height vs. how fast a person runs) into table, charts, and graphs. Describe the relationships observed from the data.
3. Methods of representing analyzed data.	b. make, read, and interpret multiple representations including tables, charts and graphs of data (NCTM Standard 10; MO 1.5, 1.8, 3.3)	• Locate historical data (such as results from athletic events, annual precipitation for the county, or population) and prepare a graphic representation. Analyze the results to make predictions for the future.
4. Similarities and differences in theoretical and experimental probabilities.	c. formulate, predict, and defend positions taken that are based on data collected (NCTM Standard 10; MO 1.2, 1.4, 2.1, 3.7)	• Prepare a set of questions and responses for a survey that will be given to fellow students to determine opinions on a current event. Distribute and collect the surveys, then analyze and display the results.
5. The appropriate use of technology.	d. analyze information and arguments that are based on data collected (NCTM Standard 10; MO 1.7, 3.4, 3.6)	• Given a media representation of data, write a narrative for the information.
	e. investigate the power of making decisions based on statistical methods and the applications of probability in the real world (NCTM Standard 10; MO 1.3, 3.2, 4.3, 4.7)	• Given data from a scientific experiment, organize the data into a symbolic or graphic representation.
	f. use computers, graphing calculators, and/or other forms of technology to enhance understanding of numbers, data, and the resulting analysis (NCTM Standard 10; MO 1.4, 2.7)	

VII. Data Analysis, Probability and Statistics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	g. develop and execute experiments or simulations to predict and determine probable outcomes (NCTM Standard 10; MO 1.2, 1.3, 3.1, 3.6)	• Randomly generate pairs of numbers with a sum ranging from 2 through 12 via number cubes, graphic calculators, or computers. Vary the sample size. Organize the results into graphical form. Describe the results.
	h. investigate sample spaces to predict probable outcomes and how these predictions affect the decision-making process (NCTM Standard 10; MO 1.3, 1.7, 1.10, 3.6)	• Given the 1-week daily attendance to an amusement park, swimming pool or water park and the corresponding weather reports, predict the daily attendance for a different week, given the 7-day weather forecast. Compare predicted results to actual results.
	i. investigate appropriate applications for experimental and theoretical probabilities (NCTM Standard 10; MO 1.7, 3.8, 4.7)	

VII. Data Analysis, Probability and Statistics

What All Students Should Know

By the end of grade 12, all students should know

1. Statistical measures of central tendency, randomness, variability, and correlation.
2. Appropriate use of theoretical and experimental probabilities.
3. The process required to design and conduct a survey or experiment.
4. The process required to analyze and present data.
5. The appropriate use of technology.

What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

By the end of grade 12, all students should be able to

a. interpret and summarize data from charts, tables, and graphs that appear in real-world situations (NCTM Standard 10; MO 1.1, 1.8)

b. apply curve-fitting to make defendable predictions (NCTM Standard 10; MO 1.4, 2.7, 3.2)

c. apply the appropriate statistical measures including central tendency, variability, and correlation to a situation (NCTM Standard 10; MO 1.2, 1.5, 3.2)

d. investigate the effects of data transformations on variability and measures of central tendency (NCTM Standard 10; MO 1.1, 1.4, 2.7)

Sample Learning Activities

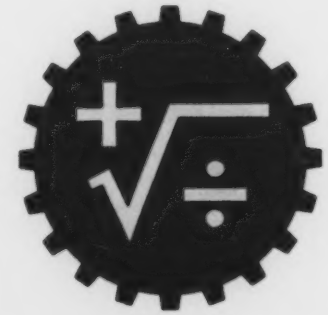
NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than data analysis/ probability and statistics.

- Investigate a municipal or county service (such as, energy, water, or trash collection) and determine usage over the past 20 years. Predict the quantity or usage 10 years from now. Analyze and describe how conservation and recycling can affect your prediction.
- Collect data from experimentation, a survey, or use published data to determine measures of center, spread, and extrema and use statistical measures to compare and make predictions about the future.
- Determine and implement methods for conducting an unbiased survey (such as a phone survey for political candidate).
- Organize data in a chart, stem/leaf, or box plot, and use technology to determine the line or curve of best fit.
- Design models to simulate relative frequency and probability of an event occurring.
- Determine the relative frequency of an event and relate it to theoretical probability.

VII. Data Analysis, Probability and Statistics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	e. investigate the concept of a random variable (NCTM Standard 10; MO 1.4, 2.7, 3.2)	• Investigate a career that uses data analysis and make a presentation based on your finding.
	f. design and interpret simulations to estimate probabilities (NCTM Standard 11; MO 1.3, 3.3, 3.6)	• Determine the odds of an event occurring.
	g. apply theoretical probability to real-world problems (NCTM Standard 11; MO 1.7, 3.8)	• Apply data collection, representation of data, and interpretation of data to solve real-world problems.
	h. apply experimental probability to real-world problems (NCTM Standard 11; MO 1.7, 3.8)	• Design and conduct a field or laboratory study by gathering and interpreting data, formulating a hypothesis, and expressing the results in a variety of presentations.
	i. collect, plot, and interpret data, including that from a discrete probability distribution (NCTM Standard 11; MO 1.2, 1.6, 3.6)	
	j. develop, interpret, and apply the normal curve in problem solving (NCTM Standard 11; MO 1.1, 3.2, 3.4)	
	k. determine and interpret maximum and minimum values within a data set, on a graph, or in a problem situation (NCTM Standard 13; MO 1.3, 2.1, 3.6)	
	l. analyze an infinite series as it relates to a limiting value (NCTM Standard 13; MO 1.6, 1.8, 3.2)	

VIII. Patterns and Relationships
Show-Me Standards, Math 4



K-12 Content Overview

Many mathematicians regard mathematics as the "science of patterns." Investigating the patterns that they find in numbers, shapes, and expressions is perhaps the most successful strategy students can use to make true mathematical discoveries. In so doing, they will be solving problems, dealing with important mathematical concepts and relationships, making and verifying generalizations, and constructing an initial understanding of a fundamental mathematical idea–the function.

All of the content strands have close interconnections, but this one is very closely tied to all of the others since an understanding of patterns can be either content or process. When the patterns themselves and their rules for generation are objects of study, they represent the content being learned. There is a very special relationship, though, between patterns and algebra. Algebra provides the language in which we communicate the patterns in mathematics.

Children become aware of patterns very early in their lives. Repetitive daily routines and periodic phenomena are all around them. In the early school grades, children need to build on those early experiences by constructing, recognizing, and extending patterns in a whole variety of contexts. Numbers and shapes certainly offer many opportunities, but so do music, language and physical activity. Children should construct their own patterns with manipulatives such as pattern blocks, attribute blocks and multilink cubes. The calculator is also a very useful tool for making the connection between counting patterns and rules of arithmetic operations.

In the higher elementary grades, input/output activities that require recognition of relationships between one set of numbers and a second set provide an early introduction to functions. In the middle grades, students begin to work with patterns that can be used to solve problems within the domain of mathematics as well as from the real world. Graphing software is extremely valuable at this level to help students visualize the relationships they discover.

At the secondary level, students are able to bring more of the tools of algebra to the problem of analyzing and representing patterns and relationships. The use of functions in modeling real-life observations also plays a central role in the high school mathematics experience. Graphing calculators and computers should be available to all students for use in line- and curve-fitting as approaches to the explanation of a set of experimental data .

VIII. Patterns and Relationships

What All Students Should Know

By the end of grade 4, all students should know

1. Mathematical ideas may be represented with visual models.
2. Mathematical symbols can be used to represent real-world situations.
3. Patterns and relationships can be represented in a variety of ways.
4. Information can be organized to look for a pattern or relationship.
5. Patterns can be geometric and/or numeric.

What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

By the end of grade 4, all students should be able to

a. create, recognize, describe, and extend a wide variety of patterns (NCTM Standard 13; MO 1.6, 1.8, 2.1, 3.3)

b. represent and describe mathematical relationships (NCTM Standard 13; MO 1.6, 1.8, 2.2, 3.3)

c. investigate the use of variables and open sentences in expressing relationships (NCTM Standard 13; MO 1.6, 1.8, 3.3)

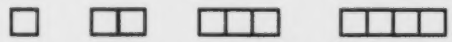
Sample Learning Activities

NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than patterns and relationships.

- Given a picture of a design in a quilt, describe all the patterns you see. Explain what could be added to the design to generate yet another pattern.

- Use square tiles to make the first four rectangles in the design below. Count the number of squares in each rectangle and the number of units around each rectangle. Write a rule you might use to find the number of squares and the number of units around for the fifth rectangle.

a.



b.

- Given a hundreds chart with a few numbers darkened on the first two rows, complete a pattern over the entire chart. Describe or classify the numbers that have been darkened over the entire chart. Extension: Given a blank number hundreds chart and counting disks, make an original pattern. Change the pattern by moving a designated number of counting disks. Describe both patterns and their differences.

VIII. Patterns and Relationships

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Using the constant function on a calculator, construct an input/output table of numbers. Describe the relationships. Graph the results. • Repeat a rhythm pattern begun by the teacher (such as clap-clap-clap-stamp-stamp-stamp-clap-clap-clap- or snap-clap-snap-clap-snap). Now create your own rhythm pattern and ask a classmate to repeat it. • Practice initiating or extending geometric patterns or codes (such as circle-square-circle-square or AB-AAB-AAAB). • Line up with six or seven of your classmates according to a pattern (such as boy/girl; short/tall). Ask other students to name the pattern(s) they see.

VIII. Patterns and Relationships

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know 1. Mathematical ideas may be represented with visual models. 2. Mathematical symbols can be used to represent real-world situations. 3. Patterns and relationships can be represented in a variety of ways. 4. Information can be organized to look for a pattern or relationship. 5. Patterns can be geometric and/or numeric.	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 8, all students should be able to* a. examine, predict, design, extend, and describe patterns and relationships (NCTM Standard 8; MO 1.6, 1.8, 2.1, 3.3) b. design and compare patterns and relationships using rules, charts, and graphs that may be constructed using technology (NCTM Standard 8; MO 1.6, 1.8, 2.7, 3.3) c. examine patterns, relations, and functions to determine how a change in the independent variable can produce a change in a dependent variable (NCTM Standard 8; MO 1.6, 1.8, 3.3, 3.6) d. apply patterns, relations, and functions to solving real-world problems (NCTM Standard 8; MO 1.6, 1.8, 3.3, 3.6) e. solve equations and inequalities (NCTM Standard 9; MO 1.6, 1.8, 2.2, 3.3)	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than patterns and relationships. • Predict the next three terms in the following sequence: -5, -8, -11, -14, ... Explain how you reached your answer. Use your model to determine the 100th term. • Make a diagram to show the maximum number of pieces of pizza made with three cuts across the center, four cuts across the center, and five cuts across the center. Find an explicit formula relating the number of cuts to the maximum number of pieces of pizza. • Collect three flowers that grow in the Fibonacci pattern. Make a sketch of the petal pattern and discuss how it is within the Fibonacci set. • List the first five rows in Pascal's triangle. Devise a pattern to symbolically write the next row in terms of the first five rows. • Apply patterns of chaos to real-world situations (for example, shore lines).

VIII. Patterns and Relationships

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 12, all students should be able to*	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than patterns and relationships.
1. Mathematical patterns and relationships may be represented in various forms.	a. compare and contrast the real number system and its various subsystems with regard to their structural characteristics (NCTM Standard 14; MO 1.6, 1.8)	• Use varied technologies or problem-solving techniques to explore the following situation: A heavy equipment operator makes an hourly wage of $15.00 per hour and a laborer makes an hourly wage of $10.00 per hour. The laborer starts preparing the job for work at 7:00 a.m. The heavy equipment operator starts work at 8:00 a.m. Each worker gets a 1-hour lunch break. Explore ways to determine the time of day each worker will have earned the same amount of wages for the day. How much money will each worker have made at the end of 8 hours? How much money will each worker have made by 5:00 p.m.? (Workers receive time and one-half for working more than 8 hours in a day.)
2. Mathematical symbols can be used to represent real-world situations.	b. represent and analyze relationships using verbal rules, tables and graphs as tools to interpret expressions, equations and inequalities (NCTM Standards 5 and 6; MO 1.6, 1.8, 2.1, 3.3)	
3. Definitions of sequences and series.	c. translate among tabular, symbolic, and graphical representations of functions and model real-world phenomena with a variety of functions (NCTM Standard 6; MO 1.6, 1.8, 2.2, 3.6)	• Use varied technologies or problem-solving techniques to explore the following situation: A dog pen is being built beside the garage. The garage serves as one side of the fence. The amount of fence purchased is 20 feet. Explore ways to determine the maximum area for the fence. What length and width will give the maximum area?
4. Trigonometric ratios of sine, cosine, and tangent.	d. represent situations that involve variable quantities with expressions, equations and inequalities (NCTM Standard 5; MO 1.6, 1.8, 3.3)	• Design a diagram for a candy box that contains five different kinds of candy. Each box has at least three pieces of candy of the same kind. No two of the same kind of candy are next to each other on the same row, on the column or on the diagonal. Could the candy box be designed in more than one way? Justify your answer.
5. Subsets of the real number system.		

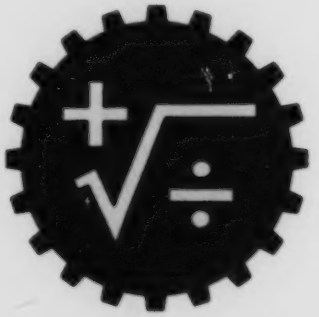
VIII. Patterns and Relationships

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	e. solve equations and inequalities (NCTM Standard 5; MO 1.6, 1.8, 2.2, 3.3)	• A summer job consists of mowing yards for 16 weeks. The first week the pay is $0.25. The pay doubles each week. At what week will the pay be more than $100.00 for the first time? What would the pay be for mowing the lawn on the 16th week? Graph your results. Do you think that the employer can complete the payment for this plan all 16 weeks? Write a paragraph to justify your answer.
	f. translate between synthetic and coordinate representation for geometric relationships (NCTM Standard 8; MO 1.6, 1.8, 2.2, 3.3)	
	g. investigate limiting processes by examining infinite sequences and series (NCTM Standard 13; MO 1.6, 1.8, 3.3)	• Do the following problem: The price of business cards are: STORE A - $30.00 for the first 200 cards and $0.05 for each additional card; STORE B - $0.10 for each card. a. For what number of cards is the cost the same at both stores? b. How many cards would you need to order so that STORE A has the best price? c. Is there ever a situation where ordering less cards than those required for the breakeven point might be justified? Write a paragraph supporting your answer.
	h. apply trigonometry to problem situations involving triangles and explore real-world phenomena using the sine, cosine, and tangent functions (NCTM Standard 9; MO 1.6, 1.8, 2.2, 3.6)	
	i. analyze effects of parameter changes on the graphs of functions using a variety of technologies to gather data (NCTM Standard 6; MO 1.4, 1.6, 2.7, 3.3)	• Use graphing technology to explore families of algebraic expressions to determine if they are linear, non-linear, quadratic or cubic.

VIII. Patterns and Relationships

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Play the Chaos Game. Draw an equilateral triangle. Label the vertices A, B, C, etc. Use a spinner or number cube to generate random numbers. On a number cube, vertex A could be represented by numbers 1 and 4. Vertex B could be represented by numbers 2 and 5. Vertex C could be represented by numbers 3 and 6. Start with any point in or around the triangle. Roll the cube or spin the spinner to determine a vertex. Measure one-half the distance between the point and the vertex. Make a dot. Generate a random number to indicate another vertex. Measure one-half the distance from the point to the vertex. Make a dot. Continue this procedure until a pattern starts to emerge. Do the activity on transparencies for the overhead projector. Put several groups of transparencies on top of the other to give a more complete pattern. Describe the pattern. (Adapted from *Making the Connections: Fractal Activities for the Secondary Classroom*, NCTM)

IX. Mathematical Systems and Number Theory
Show-Me Standards, Math 5



K-12 Content Overview

Understanding the fundamental operations of addition, subtraction, multiplication and division is central to knowing mathematics. One essential component of what it means to understand an operation is recognizing conditions in real-world situations that indicate that the operation would be useful in those situations.

Children need extensive informal experience with problem situations and language prior to explicit instruction and symbolic work with the operations Thus, informal experience with all four operations should begin in kindergarten and continue through grade four. By emphasizing underlying concepts, using physical materials to model procedures, linking the manipulation of materials to the steps of the procedures, and developing think patterns, teachers can help children master basic facts and algorithms and understand their usefulness and relevance to daily situations.

In the middle school years, students come to recognize that numbers have multiple representations, so the development of concepts for fractions, ratios, decimals, and percents, and the idea of multiple representations of these numbers need special attention and emphasis. The ability to generate, read, use and appreciate multiple representations of the same quantity is a critical step in learning to understand and apply mathematics.

By the high school level, students should come to understand and appreciate mathematics as a coherent body of knowledge rather than a vast, perhaps bewildering, collection of isolated facts and rules. Understanding mathematical structure promotes students' efficiency in investigating the arithmetic of fractions, decimals, integers and rationals through the unity of common ideas. It also offers insights into how the whole number system is extended to the rational number system and beyond.

Relationships among operations are developed over many grades. At the K-4 level, multiplication can be viewed as repeated addition, and division can be considered as repeated subtraction. However, because the multiplication of fractions and decimals is not repeated addition and division is not always repeated subtraction, students should be exposed to other interpretations of multiplication and division as well. The development of personal meaning for numbers should be reinforced in the middle grades with an extension to other numbers and notations. Student experiences should include exploration of the properties of sets of numbers. High school students should extend their meaning of number to the real number system and a recognition that still other number systems exist, such as geometry. They should have the opportunity to develop intuitive proofs of properties of operations and sets of numbers, such as closure, commutativity, associativity and geometric properties.

IX. Mathematical Systems and Number Theory

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know 1. Basic operations of addition, subtraction, multiplication and division are related to each other. 2. The concepts of factors and multiples in relation to multiplication and division.	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 4, all students should be able to* a. develop the "need" for whole numbers, integers and rational numbers, including fractions and decimals by looking for patterns and relationships to solve problems (NCTM Standard 6; MO 1.6, 3.2, 3,3) b. develop and use number operations and order relations for decimals (money) (NCTM Standard 6; MO 1.6, 3.2, 3.3) c. develop an understanding of how basic arithmetic operations are related to one another (NCTM Standard 6; MO 1.6) d. develop and use number theory concepts, including factors and multiples in problem solving (NCTM Standard 6; MO 1.6, 3.5) e. model, develop, and explain basic facts and algorithms with reasonable proficiency (NCTM Standard 8; MO 3.6)	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than mathematical systems and number theory. • Use a set of manipulatives to explain the relationship between and among the operations of addition, subtraction, multiplication and division. • Given a set of student-generated fraction or decimal cards, place the cards in order. Explain the process used. • Represent all of the possible rectangles that can be made using 1-25 manipulatives (such as color tiles or unifix cubes). Explore and discuss the relationships of numbers that have multiple representations, a single representation, and can be represented by a square. • Given a hundreds chart or 0-99 chart, search for patterns like doubles, odd numbers, even numbers, numbers with a 7 in them, numbers whose digits add 10, etc. • Given a temperature of 20° at 8:00 p.m., develop a model to represent the temperature at 7:00 a.m. if the temperature drops an average of 4° per hour.

IX. Mathematical Systems and Number Theory

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
		• Play "Oh, No, 49!" Deal 5 cards to each player. The player to the left of the dealer places a card in the center of the table, announcing the value of the card, and takes a card from the deck. The next player plays a card, adding the value of his/her card to the first card played. The player then also takes another card from the deck. Play continues, keeping a running total of cards played. Game ends when a player forces another player to exceed 49. Card values: A = 1; 2-9 = face value; 10= -10; J reverses operations (if you were adding, a jack causes the play to continue by subtracting); Q = 0; and K makes the total 49.

IX. Mathematical Systems and Number Theory

What All Students Should Know

By the end of grade 8, all students should know

1. Commutative, associative, and distributive properties.
2. Properties of zero and one.
3. Patterns may be used to describe relationships for multiples, factors, and exponents.
4. Order of operations.

What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

By the end of grade 8, all students should be able to

a. evaluate the need and applications of numbers not contained in the set of whole number (NCTM Standard 6; MO 1.1, 3.2, 3.3, 4.1)

b. develop an understanding of and explain order and relationship among integers, fractions, and decimals (NCTM Standard 6; MO 1.6, 2.2)

c. use real-world and mathematical problem situations to develop and apply number theory concepts (such as primes, factors, and multiples) (NCTM Standard 6; MO 1.6, 1.10, 3.2, 3.3)

d. realize the dynamic nature of mathematics and how different mathematical systems apply to current and developing real-world situations (NCTM Standard 6; MO 1.10, 2.5, 4.5, 4.8)

e. apply commutative, associative, and distributive relationships in computation and estimation situations. (NCTM Standard 6; MO 1.6; 3.2, 3.3)

f. recognize the connection of irrational numbers and the real world. (NCTM Standard 6; MO 1.6, 3.2, 3.3)

Sample Learning Activities

NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than mathematical systems and number theory.

- Investigate a problem situation requiring numbers not contained in the whole number set (such as figuring out the number of 70-seat buses necessary to transport 342 students or dividing a candy bar into 7 equal parts).

- Demonstrate a knowledge of order and relationship between various number sets through a problem situation. The activities could involve having each class member select a real number (including whole, natural, integer, rational or irrational numbers). Then make a number line so each person is in the proper order. Compare two similar numbers (such as 16.01 and 16.009876) by writing an explanation of each number's size and location on the number line.

- Describe, model, diagram, andor graph and the subsets of the real number set, such as whole, integer, and rational numbers. The description may include an investigation of real-life careers utilizing the various sets and an explanation of the ways the numbers are used in the real world.

- Use technology to begin an informal investigation of irrational numbers including, Pi, the square root of 2 or square root of 3 to discover where these types of numbers originated and how they are used in the real world.

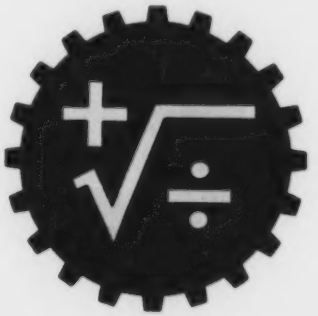
- Investigate to determine which set of numbers is larger — rational or irrational. Then justify your conclusion.

IX. Mathematical Systems and Number Theory

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may als relate to strands other than mathematical systems and number theory.
1. Properties of real numbers.	*By the end of grade 12, all students should be able to*	• Test for closure, commutative, associative, identity, inverse, and distributive properties in subsystems or the real numbers such as positive integers, integers, and rational numbers.
2. Procedures for calculator and/or computer use.	a. compare and contrast the real number system and its various subsystems (NCTM Standard 14; MO 2.1, 4.1)	• Explore triangular numbers, primes, composites, factors, the Fibonacci sequence, Pascal's Triangle, etc., to enhance understanding of number relationships.
3. Basic algebraic procedures.	b. select and apply appropriate technology as a problem-solving tool to achieve understanding of the logic of algebraic and geometric procedures (NCTM Standard 14; MO 1.4, 3.6)	• Model algebraic procedures using various technologies such as tiles, balance, or graphing calculators.
4. Geometric relationships.	c. investigate and determine similarities and differences between mathematical systems (NCTM Standard 14; MO 2.1, 4.1, 4.6)	• Use diagrams to demonstrate an understanding of the logic of conjunctions and disjunctions.
	d. extend understanding and application of number theory concepts (NCTM Standard 6; MO 1.6, 3.2, 3.3)	• Investigate sets of symmetries of geometric figures to compare their structures (for example, isosceles triangles and parallelograms).
		• Investigate the structural commonalities of the binary system as it applies to multiplication, the logical operations of "and," and electrical current.
		• Investigate geometry as a mathematical system by making and testing conjectures to arrive at generalizations.
		• Develop and illustrate the use of your own mathematical system.

MATHEMATICS 9-12

X. Discrete Mathematics
Show-Me Standards, Math 6



K-12 Content Overview

Many applications tied to the principles in business, to computer science, and to other real-world problems involve an area of mathematics called "discrete mathematics." The word *discrete* is defined as "separate or distinct." Discrete mathematics is the study of points, ideas and objects that are separate from each other or distinct. John Dossey, in his article DISCRETE MATHEMATICS: THE MATH OF OUR TIME (NCTM Yearbook, 1991) states that "discrete mathematics allows students to explore unique problem situations that are not directly approachable through writing an equation or applying a common formula."

Modeling or other forms of representation are often required to help students visualize the situation as well as other areas of mathematics. Discrete mathematics builds upon and extends the mathematics in the first nine strands of this document. Algorithmic thinking, graph theory, probability and counting techniques, mathematics of social decision-making, matrices, and recursion are all included in this strand.

Discrete mathematics promotes the making of mathematical connections, provides a setting for problem solving involving real-world applications, takes advantage of a technological setting, and provides the opportunities for critical thinking and mathematical reasoning. (Kenny, NCTM Yearbook, 1991) Through discrete mathematics, students are involved in a variety of experiences that build on previously developed mathematical concepts. Connections to real-life problem solving allow students to look at possible career choices to take them beyond the year 2000.

X. Discrete Mathematics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 4, all students should know 1. Numbers in sequence to count objects. 2. Definition of "more" and "fewer." 3. Definition of "same" and "different." 4. Definition of "shortest" and "longest."	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 4, all students should be able to* a. determine what should be counted in a set of objects and actually count the objects (NCTM Standard 6; MO 1.8) b. predict whether the set contain more or fewer of one subset than the other (NCTM Standard 6; MO 2.2) c. illustrate or explain how the subsets of objects are the same or different (NCTM Standard 3; MO 1.8) d. identify and discuss overlapping subsets of objects (Venn diagrams) (NCTM Standard 3; MO 2.2) e. create algorithms based on constructing meaning from explorations (NCTM Standards 7 and 8; MO 1.6, 3.4, 3.6)	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than discrete mathematics. • Use concrete materials (manipulatives) and/or diagrams to illustrate a variety of problem -solving situations with emphasis on thinking and common sense. Sequence then count a set of objects. Group the objects to count by 2s, 3s, 5s, and 10s. How does this relate to the concept of multiplication? How many different ways can three distinct object counters be arranged in a row? How do you know you have found all the ways? Explain or draw a diagram. • Select a method of comparison and then sort concrete materials, pictures, or diagrams to match their rule. Make comparisons, looking for alike or different, more or less, smaller or larger, etc., then write about or tell about the method of comparison selected and how each part relates to the entire set of materials. • Take concrete objects or pictures that have more than one attribute such as color, shape, or size. Then use a diagram to sort the objects using the rule that was selected.

X. Discrete Mathematics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	f. determine a path through a maze, whether a street network could be traveled going over each street one time, and the shortest distance traveling on a network of roads or streets (NCTM Standard 9; MO 2.2, 3.3, 3.4)	• Investigate algorithms in the study of mathematics to help organize and structure thinking. An algorithm is a sequence of instructions that if followed for an operation will always lead to a defined result. Use concrete materials to invent algorithms to solve addition, subtraction, multiplication, and division problems. Sequence a given set of pictures. Illustrate a sequence of instructions on how to get home from the school, or the order in which you do things when you are getting ready for school in the mornings.
	g. apply the concept of *fair division* to real-world situations (NCTM Standard 1; MO 2.2, 3.2, 3.3, 3.4, 3.7) **fair division*: the division of an object (such as food among children or an estate among heirs) in a fair way for all people involved.	• Draw a map of two routes to get from school to your home. Determine the shorter route, then talk about or write about why one way of going home is better than the other. • Arrange attribute blocks or attrilinks into a train so each piece is next to another piece that is different from it in just one way. • Given a collection of objects, sort the materials into two, three, or four sets (these sets could be placed within string loops). Record the attributes for each set on a sheet of paper. Have another group of students observe the sets and try to identify the attributes. • Explore the idea of fair division by sharing a candy bar with a friend. Divide the candy bar into two equal pieces and have the friend pick the first piece. Is this fair or not? Devise a way to divide a cake or pizza to be shared by four friends. Is the method fair? Explain your answer.

X. Discrete Mathematics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 8, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." *By the end of grade 8, all students should be able to*	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than discrete mathematics.
1. Definition and example of patterns. 2. Definition and example of tree diagrams.	a. determine and continue a pattern using inductive reasoning (NCTM Standard 3; MO 1.6, 1.8, 3.5, 4.1)	• Extend a pattern made with blocks; make a table of the numerical values to represent the pattern. Determine the 10th term and the 100th term.
3. Definition and examples of Venn diagrams.	b. look at if-then relationships to make logical deductions (NCTM Standard 3; MO 1.6, 3.5, 4.1)	• Write statements such as, "If I go to the football game Friday evening, then I will not be home." Determine the relationship of the "if" part of the statement to the "then" part of the statement.
4. Definition and examples of networks.	c. investigate tree, Venn, or student-developed diagrams as an organizing tool for problem solving (NCTM Standard 3; MO 1.8, 2.1, 2.2)	• Use diagrams as tools to find the solution of non-routine problems, such as the following: The 28 students in a homeroom class are having a pizza party. Fifteen students like pepperoni pizza and 17 students like hamburger pizza. All of the students like pepperoni, hamburger, or both kinds of pizza. How many students like both kinds of pizza?
	d. explore transportation networks (NCTM Standards 2, 4, and 12; MO 2.2, 3.2, 3.3, 3.6)	• Use a local map that shows a portion of the area where you live. Design a route to deliver pizza to six of your friends. Design the route so that the driver will drive the least possible distance.
		• Design a tournament bracket for a contest in the intramural program for your school by using a diagram. Do certain numbers work better for making a tournament bracket than others? If certain numbers work better, explain their characteristics.

X. Discrete Mathematics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
By the end of grade 12, all students should know	NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."	NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than discrete mathematics.
1. Concepts of geometry.	*By the end of grade 12, all students should be able to*	
2. Concepts of probability and statistics.	a. explore and solve application problems involving graph theory (airline routes, circuits, paths, connecting roads, coloring a map, etc.) (NCTM Standard 12; MO 1.6, 1.8, 2.2, 3.2, 3.3, 3.6)	• Design a shopping complex, a city park, or a trail containing intersecting paths or roads. The design must allow visitors to walk or drive through the entire area without going over the same trail or road twice. Visitors may go through an intersection more than one time with trails or roads previously traveled.
3. Characteristics of exponential growth.	b. use tree, Venn, or student-developed diagrams as problem-solving tools (NCTM Standard 12; MO 3.2, 3.3, 3.6)	• Use diagrams as a way to visually account for all of the possibilities in a counting situation. Sample problem: A restaurant near your school offers dinners with four different meats and three different vegetables. The dinner contains one meat choice and two vegetable choices. How many different dinners could be ordered?
4. Cause-and-effect.	c. use concepts from logic and/or truth tables to recognize valid and invalid arguments (NCTM Standard 3; MO 2.2, 3.5)	
5. Whole-to-part.	d. explore applications from counting techniques such as Pascal's Triangle, permutations, combinations, and Fibonacci sequence (NCTM Standard 12; MO 1.6, 2.2, 3.6)	• Research how the apportionment in the House of Representatives is determined. Determine how it is possible for a state to gain population by the last census and lose numbers of representatives. Illustrate the mathematics involved in the apportionment process that would allow this to happen. Write a paragraph to explain the results of your research.

X. Discrete Mathematics

What All Students Should Know	What All Students Should Be Able To Do	Sample Learning Activities
	e. investigate the concepts of *game theory** (NCTM Standard 1; MO 3.2, 3.7, 3.8) **game theory*: selecting the best strategies in order to achieve the most favorable outcomes. Games are defined as having two or more players with conflicting interests. f. explore concepts from election theory (NCTM Standard 1; MO 3.2, 3.7, 4.2, 4.3) g. investigate different approaches to apportionment and fair division, then explore their applications (e.g., division of property in estates, apportionment in the House of Representatives) (NCTM Standard 1; MO 3.2, 3.3, 3.7, 4.3) h. use the concept of recursion in mathematics to solve application problems (e.g., compound interest, depreciation, radium decay, maximum storage in the least amount of space, fractals) (NCTM Standard 12; MO 1.8, 2.2, 3.2, 3.7)	• Write a mathematical (recurrence) relationship that may be used to determine the amount of money in the savings account described below after n months. Determine the amount of money in the savings account at the end of 1 year. James has $1,000.00 in a savings account when he finishes college and gets his first full-time job. He has included in his financial plan a monthly $50 deposit in a savings account. The annual interest rate is 4.8% compounded monthly. Complete the table below. n (in months) t_n 0 $1,000 1 1.004(1000) + 50 = $1054 2 3 . . . 12 . . . n

SUGGESTED PRACTICES FOR TEACHING AND LEARNING MATHEMATICS

Basic Knowledge of the Concepts of Mathematics

Schools must teach a well-rounded core of mathematics to all students. In order to build a knowledge-base for mathematics, schools should deemphasize the use of pencil and paper assignments and incorporate additional active learning through technology. The use of concrete materials helps students gain greater understanding of basic mathematics and enables them to apply the skills acquired. The need to know and be able to do mathematics requires educators to utilize a problem-solving format. The "know" refers to a knowledge of basic mathematics necessary for students to be successful problem solvers. This includes, but is not limited to, memorization of basic facts. Through investigating problem situations, students are enabled to make connections to previously learned mathematics, other subject areas, and real-world applications, thus increasing their understanding of mathematics and their ability to reconstruct the mathematics for given situations. Mathematics is more than the memorization of isolated facts and procedures. Students must have the ability to set up problems with the appropriate operations and use those basic operations to solve problems in a variety of situations.

Use of Technology

With the rapid growth of the uses and types of technology, there is an increasing need for understanding mathematics. All educators and patrons must deal with the availability of appropriate and accessible electronic and manipulative devices. For this document, **technology will be defined as any device that aids a student in viewing concrete or abstract ideas. This includes electronic devices (such as computers, calculators, graphing calculators and laser disks) and hands-on manipulatives (such as geoboards and counting tiles).** In the past, technology has seemed to be at odds with mathematics education. Most districts have traditionally used technological tools only after mastery of basic facts. Research has demonstrated that appropriately used and applied technology improves student comprehension and understanding. This has led both state and national organizations to recommend classroom use of technology for all grade levels.

Industry has changing needs and requirements for its workforce. Technology is the catalyst for this change. Technological devices must be used to motivate students to think, work, and become competent citizens. Technology frees educators to propel their students to higher levels of thinking and understanding by decreasing time spent reviewing low-level comprehension facts. Technology does not eliminate the need or use for arithmetic facts, but leads students to make connections between mathematics and the real world, thereby improving knowledge and understanding of mathematics.

Society is bombarded with information. Many people feel uncomfortable analyzing the options and sorting information for a reasonable conclusion. Technology provides educators and students with options to collect and analyze data to enhance problem solving. Integrating technological devices into instruction provides students with the opportunity to make connec-

tions to improve the understanding of concrete and abstract concepts. Technology enhances methods of communication.

As technology evolves, school districts should be expected to remain current. Our society can no longer afford to train students with outdated equipment. Technology is not merely a need in mathematics, but one necessary for all curricular areas. If appropriately utilized, technology will allow students to become aware of the connections between academic studies and real-world problem situations, thereby improving understanding and recognition of the importance of learning.

Connections and Active Learning

Current research on learning supports the belief that learners must be at the center of their own learning; that is, learners must be actively engaged in the process of exploring, analyzing and understanding the world they are attempting to learn about. Supporting this view of learning are several important characteristics of learners and learning:

1. Learning starts with the learner's own ideas and theories about the world.
2. Individuals build new knowledge upon previous personal experiences and understandings.
3. Each individual brings different interests, skills, attention, attitudes, and knowledge to a learning experience.
4. Individuals learn when they are active and engaged in firsthand experiences.
5. Learners need to view learning as the ongoing process of extending and fine-tuning their ideas.
6. Learners must recognize the value of language and social interaction as important tools of learning.
7. Learners need clear expectations.

In mathematics, active learning involves students in activities whereby they investigate situations and formulate and verify ideas in order to devise solutions for a problem. Classrooms involved in active learning allow students to move from less interesting styles of learning driven by isolated facts to more exciting styles promoting greater understanding of mathematical content. The connections students make through active learning facilitate improvements in classroom instruction. Students are encouraged to study a main idea while applying knowledge from several subjects at once. Given the freedom to utilize hands-on activities, students get involved with the learning rather than passively sitting and waiting for the learning to occur. Hands-on learning also allows students to connect the concrete level (objects), the semi-concrete level (pictures), the semi-abstract level (charts, graphs and tables) and the abstract level (symbols) of mathematics to each other. Writing and justifying ideas verbally, symbolically and graphically support the learners in clarifying their own ideas. The process of sharing their beliefs and in turn hearing others enhances learning, since putting ideas into words and other forms forces students to clarify their own understanding of concepts, ideas and processes.

Teachers must play an active role in creating rich environments through planning and taking advantage of teachable moments. Teachers also must create safe environments for students to explore, test and conjecture about mathematics. A constructive, active view of the

learning process is reflected in the way much of mathematics is taught. Thus instruction should vary and include opportunities for:

- appropriate project work
- small group and individual assignments
- discussions between teacher and students and among students
- practice on mathematical methods
- exposition (explanation) by the teacher (NCTM, 1989)

Problem Solving

Students should create and examine interesting problem-solving situations. As they investigate a situation, the mathematics needed may be reviewed or introduced. Problem solving should take a variety of forms. Interesting problems may have a wide range of possible solutions. The teacher should guide the students to look at problem solving in several different ways and to utilize technology, including manipulatives, calculators and computers. According to NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), "The problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand."

Communication and Reasoning

Students should examine and discuss problem situations to select the most appropriate methods of computation: mental math, estimation, paper/pencil, or technology. Teachers should use discussion to motivate students to:

- think and talk about mathematics by asking questions
- stimulate students to ask questions
- help students work together to make sense of mathematics
- help students to rely more on themselves to determine whether something is mathematically correct
- and help students to learn to reason mathematically (NCTM, 1991)

Investigation of problem-solving situations allows students to think before devising a solution and to extend their knowledge beyond a solution. Problem solving and explorations allow students the opportunities to analyze, conjecture, generalize, extend, evaluate and reflect upon the mathematics content studied. (See NCTM Addenda Series, 5-8, Patterns and Functions.) Students must believe that mathematics makes sense.

Classroom Climate

Woven through the *National Council of Teachers of Mathematics Professional Standards for Teaching Mathematics* (NCTM 1991) are five major ideas which may represent shifts in the environment of mathematics classrooms. Mathematics classrooms need to shift toward:

- mathematical communities – away from classrooms as simply a collection of individuals;
- logical and mathematical evidence as verification – away from the teacher as the sole authority for right answers;
- mathematical reasoning – away from merely memorizing procedures;
- conjecturing, inventing, and problem solving – away from an emphasis on mechanistic answer-finding;
- connecting mathematics, its ideas, and its applications – away from treating mathematics as a body of isolated concepts and procedures. (NCTM, 1991)

Assessment

Students should expect assessments to match the methods of instruction. As learning becomes more active, so should assessment strategies. Assessing the depth of a student's knowledge and understanding of mathematical topics requires more than utilizing homework, quizzes and tests. Mathematics educators should be encouraged to use multiple methods of "getting at" student abilities. Assessment tools should also include: writings, portfolios, interviews, journals, projects, investigations and teacher observations.

Using a balanced combination of assessment tools allows students to demonstrate their creativity, ability and understanding. Varying methods of assessment provides "real-world" experiences in *producing, explaining, and demonstrating* possible results. Through enhanced assessment, growth opportunities exist for all students to attain knowledge in, and value capabilities with, mathematics.

The proposed changes in the mathematics classroom are intended to empower students to be responsible for their own learning and develop confidence in their ability to do mathematics. With opportunities to be active participants in the education process, students will be prepared to become life-long learners.

VIGNETTES FOR SUGGESTED TEACHING PRACTICES

Vignette 1*

Three days remained until the beginning of school. Sharon was sitting in her classroom, leafing through materials from the summer school class she took as part of her master's program at the nearby college. She had really enjoyed the course, which included a stimulating mix of new ideas, opportunities to experiment, and time for the teachers enrolled in the course to think and talk with one another. Former classroom teachers themselves, the two professors who team-taught the course seemed sensible and realistic, and yet they had a different orientation to mathematics teaching.

The course had been exactly what Sharon needed, for she had finished the previous school year feeling vaguely dissatisfied with her mathematics teaching. After 5 years of teaching, she had become able to manage her classroom effectively, to cover the required curriculum, and to incorporate some neat supplementary activities. Sharon had received an excellent evaluation from her principal, Mrs. Jones. The principal, however, had little mathematics background and her comments were always focused on management issues. They did not address Sharon's growing questions about her mathematics teaching.

Her sixth graders typically did quite well on the district's mathematics test. Still Sharon was troubled about her students' participation in, and success with, mathematical reasoning and problem solving. Her concerns were based on her observations that some groups of students were reluctant to participate in class discussions. She also observed that some groups of students did not go on to take algebra in the eighth or ninth grade. She wanted to do something to affect these patterns of participation. In general, she felt that her students lacked both confidence and skills with mathematical reasoning and problem solving. If she gave her students word problems to do, for example, they often gave up easily or came up with answers that made no sense at all.

In the summer class, the teachers had become familiar with the *Curriculum and Evaluation Standards*. The professors had also planned sessions in which Sharon and the other teachers had opportunities to engage in mathematical activity in ways they had never experienced before. Class members found themselves – to their surprise – wrapped up in the problems, excited about trying to convince one another of their solutions, and genuinely interested in alternative pathways and approaches.

Now, with the start of school just a few days away, Sharon was determined to begin forging a new approach in her own teaching. She realized that what she had in mind was quite different. It involved not just new techniques, but a new way of thinking about mathematics and mathematics teaching. Suddenly she worried: Would Mrs. Jones understand that her students might not be sitting so quietly and listening just to her so much this year? Did she realize that they would be – she hoped – talking more, giving reasons for their answers, coming up with new ideas and questions? Sharon hoped that she could explain to her principal that she was going to be experimenting a bit as she tried to change the way she taught mathematics but that she was not sure what form this would take as the year unfolded. She hoped that Mrs.

*Adapted with permission from ***Professional Standards for Teaching Mathematics*** (NCTM, 1991), pp. 11-12.

Jones would support her efforts and take them into account when she evaluated her this year.

Sharon then started thinking about how she was going to explain to parents about the mathematics program. She knew that they would be expecting their children to be given a placement test and then started in textbooks on the basis of their performance on those tests. Sharon was thinking about working with the class more as a whole group than she had in the past. She realized, too, that she would be having the children do fewer traditional worksheets and so the parents would be getting less standard written computational work sent home. The written work would probably look pretty different from previous years, too. Last year, for example, on the first day of school, she had given a sheet with 72 mixed computational exercises to see where the students were. This year, she smiled, she was still going to find out a lot, probably MORE, about what they knew, but her strategy would be very different.

Sharon decided that in addition to writing a letter to parents about what she was trying to do (in which she would refer to the *Curriculum and Evaluation Standards*, she would have a parent meeting about mathematics — maybe in October. She considered the possibility of demonstrating and explaining to the parents some of the mathematical reasoning and problem-solving activities their children were doing. She realized that many of her parents worked the night shift at the local plant, and so she decided to do a breakfast meeting one day. Maybe she would even invite them to stay and watch a mathematics lesson afterward. That would give them a feel for the nature of the mathematics class, and if she could think of a good example, it would be great if she could get the parents hooked into the problem, too. That way, maybe she could demonstrate the kind of understanding and discourse she was trying to foster and how she was going about it.

Sharon sat back with a sigh and stretched. There was a lot of work to do. Still, she was excited at the prospects that lay ahead. But she needed to take a break from all this thinking. With one last gulp of strong lukewarm coffee, she arose and returned to the task of putting up her bulletin boards.

*Adapted with permission from *Professional Standards for Teaching Mathematics* (NCTM, 1991), pp. 11-12.

Vignette 2*

Mrs. Martinez and Mr. Golden, who have teamed up to teach eighth grade, divide their students into groups of four. They challenge their students to show why the text says that division by zero is "undefined." The teachers want their students to know why they can't divide by zero. In the past, that was all the students had learned. Once the students figure out why division by zero is undefined, they are to prepare something that they could use to justify their explanation to the rest of the class.

Mrs. Martinez suggests that the calculator may be a useful tool for this problem. "Making up some kind of story problem for a situation that involves division might be helpful for others," adds Mr. Golden. The two teachers have arranged their large classroom so that calculators, graph paper, Unifix cubes and base ten blocks, felt-tip markers and blank overhead transparencies, rulers, and other materials are out where students can freely use them. This facilitates the use of a variety of tools. Students are encouraged and expected to make decisions about which tool to use. Several students are, in fact, preparing overheads to display their conclusions about division by zero. Others are excitedly punching calculator buttons.

"The answer keeps getting larger and larger!" exclaims a pair of girls as they watch the results obtained by successfully dividing 4 by smaller and smaller divisors with the calculator.

"Why is that important?" asks Mrs. Martinez as she watches over one girl's shoulder.

"Well, because each of the numbers we are dividing by is getting closer and closer to zero but isn't zero."

"Maybe you could make a graph to show what you are finding," suggests Mrs. Martinez.

Mr. Golden finds two students slouching sullenly in their chairs behind the room divider. "We don't understand what to do," grumbles one.

Sitting down next to them, Mr. Golden begins, "Let's see if I can help. You are trying to figure out what the special problem is in trying to divide by zero. Maybe you can use some things you already know about division. How do you know that 8 divided by 2 is 4? How could you prove that if someone challenged your answer?"

The students look at him disbelievingly. He waits. Then one says, "Well, I'd just say that 4 times 2 is 8 so 8 divided by 2 has to be 4."

"Can that help you at all with this problem?" asks Mr. Golden. He stands up.

The two students look at one another and then, sitting up a bit, begin talking. "Well, that doesn't work if you take 8 divided by 0," Mr. Golden hears one saying as he walks away.

*Adapted with permission from ***Professional Standards for Teaching Mathematics*** (NCTM, 1991), pp. 53-54.

Vignette 3*

Mr. Cohen's class of high school students is working in small groups on projects that involve collecting, organizing, and interpreting data. Before beginning, they had a discussion about different possibilities for their projects. They decided on questions that they would like to pursue, such as finding the average number of hours per week that high school students work.

One group, having read an article in the newspaper on changes in the popularity of first names in the last 65 years, has decided to investigate the most common boy's and girl's names among students their age in the city. Is John still the most common boy's name, as it was in 1925, 1959, and 1976? They are curious about what is happening with girls' names, since the popular names seem to change more often. Because their community is ethnically diverse, they also wonder how that affects the pattern of names.

Mr. Cohen works his way around to the different groups, listening, making suggestions, and verifying that the group members are listening and working together. The group that is working on the names study has decided to sample the high school population in the city and is discussing the best way to design the collection of information.

John: "Let's choose three of the high schools and then write to them and ask for a list of the students enrolled in the school. We can take our sample from those lists at random".

Jenny: " But how will we pick the three schools? And why is three a good number to pick?"

John: "It seemed like enough out of all the schools in this city if we were careful to include one of the schools that has more kids from different backgrounds, because we want to make sure our sample has lots of different kinds of names."

Anna: "I think we should try to figure out about how many high school kids there are in the whole city and then pick a size for our sample based on that information."

John (nodding): "I guess that makes sense. How are we going to figure that out, though?"

Maria: "And then how big would our sample have to be to be big enough? We want to be pretty sure our sample tells us something about all the kids in our high school."

Mr. Cohen is standing by the group. He says that their discussion so far is productive, that they are dealing with some important questions for their project. He suggests a source that might help them think about the question of how many students they need to have in their sample. He also tells them that the administration office would have a list of all the high schools and how many students attend each of them. He asks if they would like him to call and ask for that list. They say that they would. Then he asks what they are going to do once they get the list.

Vignette 4*

During a geometry unit in Ms. Warrick's pre-algebra class, Jamie posed a question that seemed worth investigating. Jamie explained she knew a stop sign had eight sides, but while walking to school she noticed a stop sign's shadow could have fewer sides.

Jamie wondered out loud, "Can a stop sign's shadow have more than eight sides?"

Ms. Warrick saw an opportunity for her students to apply and extend their knowledge of geometry. Ms. Warrick divided her class up into teams to investigate a modified version of Jamie's question. She asked the teams to find how many sides are possible for a stop sign's shadow. After the class rearranged itself into the assigned teams, Ms. Warrick provided time for the entire group to ask general questions.

Suzanne exclaimed, "I see how the shadow could be stretched out!"

Ms. Warrick asked what effect might "stretching out" have.

"Maybe more than eight sides are possible," Warren offered.

David asked, "How in the world can the shadow have more than eight sides?"

Ms. Warrick suggested several surprising results might be possible. Jimmy wondered, "How can we reflect shadows?" The class brainstormed a number of ways to model the sun and the stop sign's shadow (e.g., flashlight, overhead projector, going outside).

David asked, "Does it make a difference on the type of surface the shadow lands on?"

"What do you mean by 'type of surface'?" Ms. Warrick asked for clarification.

David explained, "When the shadow goes over the curb, or edge of the reflection surface to more than one level."

Ms. Warrick nodded and asked the class what difference this idea might make.

Cindy responded, "The number of sides a shadow has might be affected by the two different levels ... oh, wait ... that's the same as two different planes! Maybe more than one plane will affect the number of sides."

Observing the class engaged in the problem, Ms. Warrick ended the class discussion by suggesting some groups might want to consider the "more than one plane" issue as their required extension to the problem. She also suggested looking at other polygons to determine possible patterns.

Ms. Warrick reminded the class that while they are working in teams, each student is responsible for his/her own individual written explanation and conclusions of the team's findings.

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Phone: 1-800-235-7566
FAX: (617) 863-1202

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Factors and Multiples
Probability
Similarity and Equivalent Fractions
Spatial Visualization

Nebraska ETV Network. 1994-1995. *Math Vantage.* Lincoln, Nebraska: GPN.
Phone: 1-800-228-4630
FAX: (402)472-2007

Math Vantage cooperative learning materials with video materials and print materials includes activities, explorations and questions. Theprogram was developed for pre-algebra students in grades 6-9.

PATTERNS UNIT:
Discovering Patterns, Patterns with Ten, Sequences and ratios, Tessellations/Trasformations, Networks, Paths and Knots.

SPATIAL SENSE UNIT:
2D Visions in a 3D World, What's Your Angle?, Trusting Triangles, A World of quadrilaterals, Circles, Curves and Spheres, Containers: Surface Area and Volume

DATA ANALYSIS UNIT:
Data: How Do You Get It?, Data: How Do You Show It?, Data: What Does it Mean?, Chances Are

Real World Math Educational Products
PO Box 5561
Whittier, California 90607
Phone: 1-800-621-7868
FAX: 1-800-621-7868

Real World Math Resource Book, 1994
The Building Game
The Stock Market Game
The Real Estate Game
The California Vacation
The Design Game

The House that Math Built- Four computer disks, 1994

The following resources are available from:
National Council of Teachers of Mathematics
19016 Association Drive
Reston, VA 22091-1593
(703) 620-9840
(800) 235-7566

Curriculum and Evaluation Standards for School Mathematics
Professional Standards for Teaching Mathematics
Assessment Standards for School Mathematics
Kindergarten Book: Addenda Series, Grades K-6
First Grade Book: Addenda Series, Grade K-6
Second Grade Book: Addenda Series, Grades K-6
Third Grade Book: Addenda Series, Grades K-6

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Fourth Grade Book: Addenda Series, Grades K-6
Fifth Grade Book: Addenda Series, Grades K-6
Sixth Grade Book: Addenda Series, Grades K-6
Geometry and Spatial Sense: Addenda Series, Grades K-6
Making Sense of Data: Addenda Series, Grades K-6
Number Sense and Operations: Addenda Series, Grades K-6
Patterns: Addenda Series, Grades K-6
Dealing with Data and Chance: Addenda Series, Grades K-8
Developing Number Sense in the Middle Grades: Addenda Series, Grades 5-8
Geometry in the Middle Grades: Addends Series, Grades 5-8
Measurement in the Middle Grades; Addenda Series, Grades 5-8
Patterns and Functions: Addenda Series, Grades 5-8
Understanding Rational Numbers and Proportions: Addenda Series, Grades 5-8
Algebra in a Technological World: Addenda Series, Grades 9-12
Connecting Mathematics: Addenda Series, Grades 9-12
A Core Curriculum—Making Mathematics Count for Everyone: Addenda Series, Grades 9-12
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Investigations is an activity based elementary mathematics program that deeply involves students in number concepts, data, and geometry.

Watkins, Landwehr and Anne E., 1986, *Quantitative Literacy Series,* Bell Telephone Laboratories, Dale Seymour Publications
Exploring Data, Grades 7-12
Exploring Probability, Grades 8-12
The Art and Techniques of Simulation, Grades 8-12
Exploring Surveys and Information From Samples, Grades 8-12

YEKTTI CARDS, 1996, A deck of 32 cards for use with Sorting; Groups and Graphs, Dale Seymour Publications

APPENDIX A

EXAMPLES OF QUALITY STUDENT WORK

The right-hand column of these Frameworks lists many suggestions for classroom activities. Simply listing activities, however, may not communicate in a concrete way what constitutes quality student work in mathematics. This appendix addresses that need by providing a few examples of quality student work to illustrate what is possible.

There are three limitations to this appendix. First, only a few examples of student work are provided because reviewers advised the authors not to make the framework too long. Second, items in this appendix will not cover *all* of the many important topics Missouri mathematics teachers teach. Finally, some of the examples have imperfections, maybe a spelling error or two, a computational error, or a minor error of fact. We included the examples with their mistakes not because we want to promote sloppy spelling, computational errors, or factual misinformation, but because the examples are real products of students who possess other strong redeeming qualities.

Despite those limitations, the examples shown should enable readers to visualize what Missouri students should be able to do with regard to the Show-Me Standards in mathematics.

ELEMENTARY MATHEMATICS ACTIVITY

"Constructing a Polyhedron" *

Task

After looking at a poster that showed several polyhedra, students were asked to choose a polyhedron to construct. The main choices included the regular octahedron (a figure with eight equilateral triangles as faces), the regular icosahedron (a figure with 20 equilateral triangles as faces), and the cubodahedron (a figure with six squares and eight triangles as faces).

Show-Me Knowledge Standards Addressed

MA 2 geometric and spatial sense involving measurement (including length, area, volume), trigonometry, and similarity and transformations of shapes

MA 4 patterns and relationships within and among functions and algebraic, geometric and trigonometric concepts

Show-Me Performance Standards Addressed

1.1 develop questions and ideas to initiate and refine research

1.4 use technological tools and other resources to locate, select and organize information

1.6 discover and evaluate patterns and relationships in information, ideas, and structures

2.1 plan and make written, oral, and visual presentations for a variety of purposes and audiences

2.4 present perceptions and ideas regarding works of the arts, humanities, and sciences

3.1 identify problems and define their scope and elements

3.2 develop and apply strategies based on ways others have prevented or solved problems

4.1 explain reasoning and identify information used to support decisions

4.5 develop, monitor, and revise plans of action to meet deadlines and accomplish goals

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Student Work

Frist thing I did was make the SNUB ICOSIDODECAHEDRON. Then When I was done making it I notice some stuff. One thing I notice was that the faces on ma snub icosidodecahedron were pentigons and triangles. And faces means sides. Then I notice that the vertex was 4. The vertex means the points that met togethere. An example is on the next page. Then I notice that all around the pentigon is triangles and that there are 5 that fit around a pentigon. Then I notice that a pentigon never touch another pentigon. And to make it you had to make 178 triangles and 12 pentigons to make the Snub Icosidodecahedron. But I notice some of that stuff I had to know before I started.

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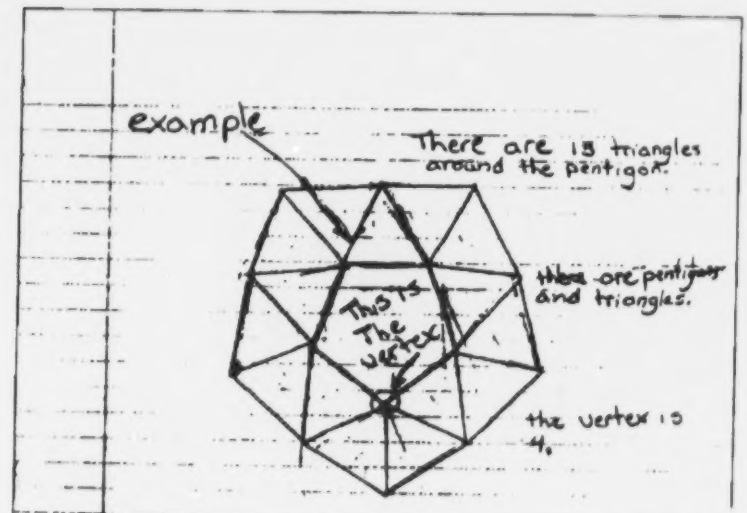

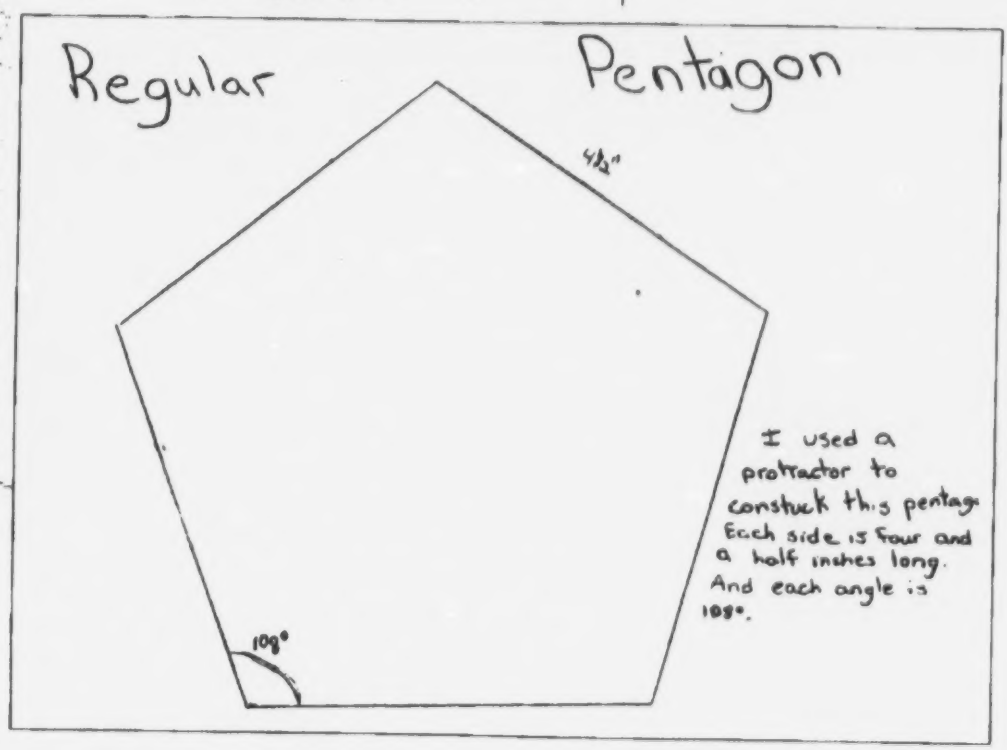
Regular Pentagon
4 1/2"
I used a protractor to construct this pentagon. Each side is four and a half inches long. And each angle is 108°.
108°

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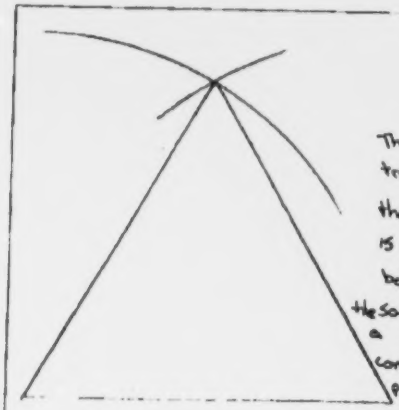
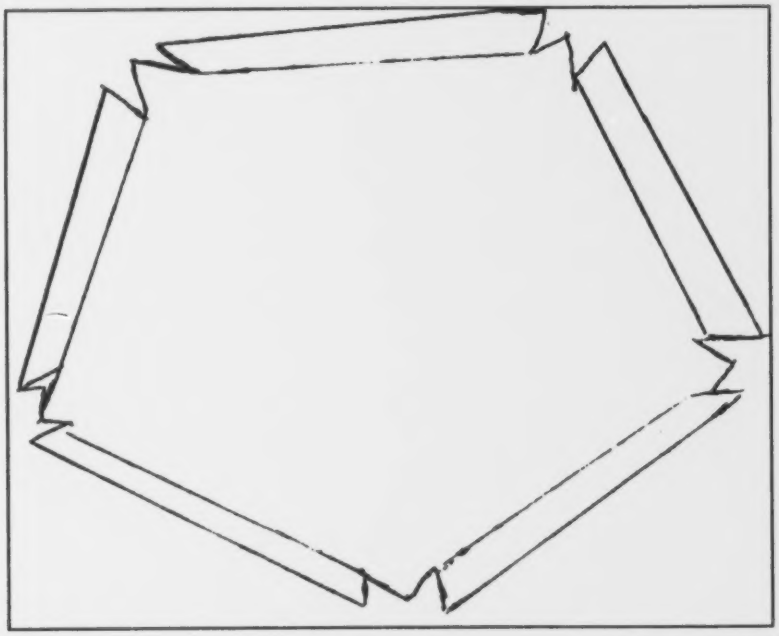


The side of my triangle is 4½' because the side of the pentgon is 4½' and the side of both parterns had to be the same. So first I made a 4½'. Then I took a compass and put the point on the end of the 4½' line I made Then I made the compass 4½' wide. Then put it on the end of the line and made a mark. Then I put on the other end and made a mark across it and it made a x. Then with the middle of the x I drew a line to one end of the 4½' line. And Then I did the same to the other side. And thats how I made my triangle.



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Commentary

The student whose response is shown wanted specifically to make a "snub icosadodecahedron," a shape considered too complex for elementary students. The student agreed to receive additional instruction and spend additional hours beyond that other students were expected to spend. This student worked extensively with polygons and successfully constructed a highly complex polyhedron made of triangles and pentagons. The work helps to illustrate what it means for an elementary student to go beyond the standard in some areas.

Completing the three dimensional model of a snub icosadodecahedron shows the depth of the student's working knowledge of geometry and measurement concepts. The steps of constructing pentagons, even with teacher assistance, and the concept and construction of the snub icosadodecahedron are more than one would expect. The student shows a high degree of manual skill by completing such a complex model. Still, in the writing, some terms are referred to incorrectly, such as "vertex." The student conveys knowledge about degrees, precise angles, pentagons, and icosadodecahedra.

The student misspells several words in the written portion of the work. This was a class assignment and not intended to be further edited for spelling or grammar. The focus of the assignment was the construction. Readers who build this figure will discover that there are actually 80 triangles, not 78, as the student reports.

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ELEMENTARY MATHEMATICS ACTIVITY

"How Many Handshakes?" *

Task

Five people enter a room and introduce themselves to each other. If everyone shakes everyone else's hand just once, what is the total number of handshakes that occurred?

Show-Me Knowledge Standards Addressed

MA 1 addition, subtraction, multiplication and division; other number sense, including numeration and estimation; and the application of these operations and concepts in the workplace and other situations
MA 3 data analysis, probability and statistics
MA 5 patterns and relationships within and among functions and algebraic, geometric and trigonometric concepts
MA 6 discrete mathematics (such as graph theory, counting techniques, matrices)

Show-Me Performance Standards Addressed

1.1 develop questions and ideas to initiate and refine research
1.6 discover and evaluate patterns and relationships in information, ideas, and structures
2.1 plan and make written, oral, and visual presentations for a variety of purposes and audiences
3.1 identify problems and define their scope and elements
3.5 reason inductively from a set of specific facts and deductively from general premises
4.1 explain reasoning and identify information used to support decisions

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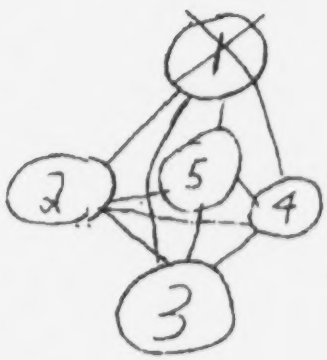
Student Work

How Many Handshakes?

Five people enteir a room and introduce themselves to each other. If everyone shakes everyone else's hand just once, what is the total niumber of handshakes that occurred.



number 1 shakes 4 people
number 2 shakes 3 People
number 3 shakes 2 People
Number 4 shakes 1 People
and number 5 shakes 0 People

4 + 3 + 2 + 1 = 10

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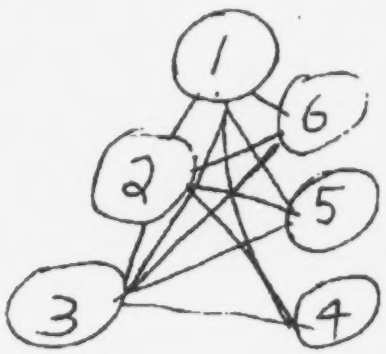
At first I made a chart using five bubbles, numbering each. I did this to find the answer number one would shake 4, not shaking his own hand. I then drawed four lines to 4, 3, 2 and 1. Then I did the egzact same routine on 4, 3 and 2 and 1. I did not shake any ones hand. Then I counted the lines and added them and it equaled 10. That is how many hand shakes in all.



1 shook 5 People hands
2 shook 4 People hands
3 shook 3 People hands
4 shook 2 People hands
5 shook 1 People hands
6 shook no hands

5 + 4 + 3 + 2 + 1 = 15

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I noticed a pattern. Since you don't have 6 shake any hands, youwrite 5, then add 4 then 3 then 2 then 1 and then it equals 15. That pattern helps me to realize not to use a chart on 7 because I can use my way like I did on the pattern. If there where 7 people I would just have to make a math sentence like this:

$$6+5+4+3+2+1=21$$

A way you can always figure it out, without a chart, is by using a math sentence Starting at the highest of your number or the lowest, going up or down then adding all of them together. Remember not to include the person who can't shake his own hand. For exsample if you had 30 people You might start with 29 because 30 can not shake his own hand. Then you would count down to 1. Then add all the numbers together and that would equal the correct answer.

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Commentary

To solve this problem, the student needs to come up with a strategy that helps order or organize the possibilities without either repeating or overlooking separate, independent handshakes. In the process of deciding on and applying an effective strategy, the student may uncover and in some way represent the mathematical essence of the problem: that there is a functional relationship (of the number of handshakes to the number of people) based on a non-linear pattern.

The student response shown includes a clear and effective approach that is modeled in two different ways and that arrives at a correct answer. The student then uses the successful problem models to go beyond the original question to solve similar problems, finally making a general rule about all such problems. The sample provides evidence for explaining in ways that make sense because it defends the reasoning and explains the solution and makes connections between concepts in order to solve problems. The student recognizes that five people each shaking each other's hands generate not five but ten handshakes; that six people generate not 12 but 15, etc. and is able to extend the non-linear iterations based on the rule generated by the student.

A close inspection of the student's diagrams reveals certain ambiguities and inconsistencies. For example, it is not always clear who shakes whose hand how many times. At other times, the syntax of the student's narrative is somewhat confusing. The response, however, is clearly good enough for an elementary student and the meaning is apparent if the reader is careful and patient. The student misspells several words in the written portion of the work. This was a class assignment and was not intended to be further edited for spelling or grammar.

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ELEMENTARY SCHOOL MATHEMATICS ACTIVITY
"Favorite School Subjects"

Task

Using a table with given data, determine the favorite subject of 10 students.

Show-Me Knowledge Standards

MA 3 data analysis, probability and statistics

Show-Me Performance Standards

1.2 conduct research to answer questions and evaluate information and ideas
1.6 discover and evaluate patterns and relationships in information, ideas and structures
1.8 organize data, information and ideas into useful forms (including charts, graphs outlines) for analysis presentation
2.1 plan and make written, oral and visual presentations for a variety of purposes and audiences
4.1 explain reasoning and identify information used to support decisions

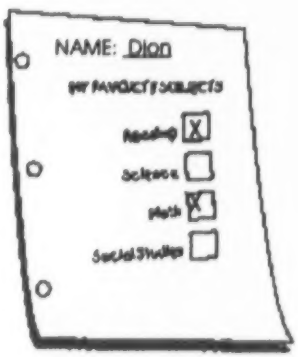
Student Work

Session 3

Directions

Show all of your work and write your answers directly in this booklet.

You want to find out which school subjects your friends like best. You asked 10 of your friends to fill in a form like this one to show their favorite subjects.



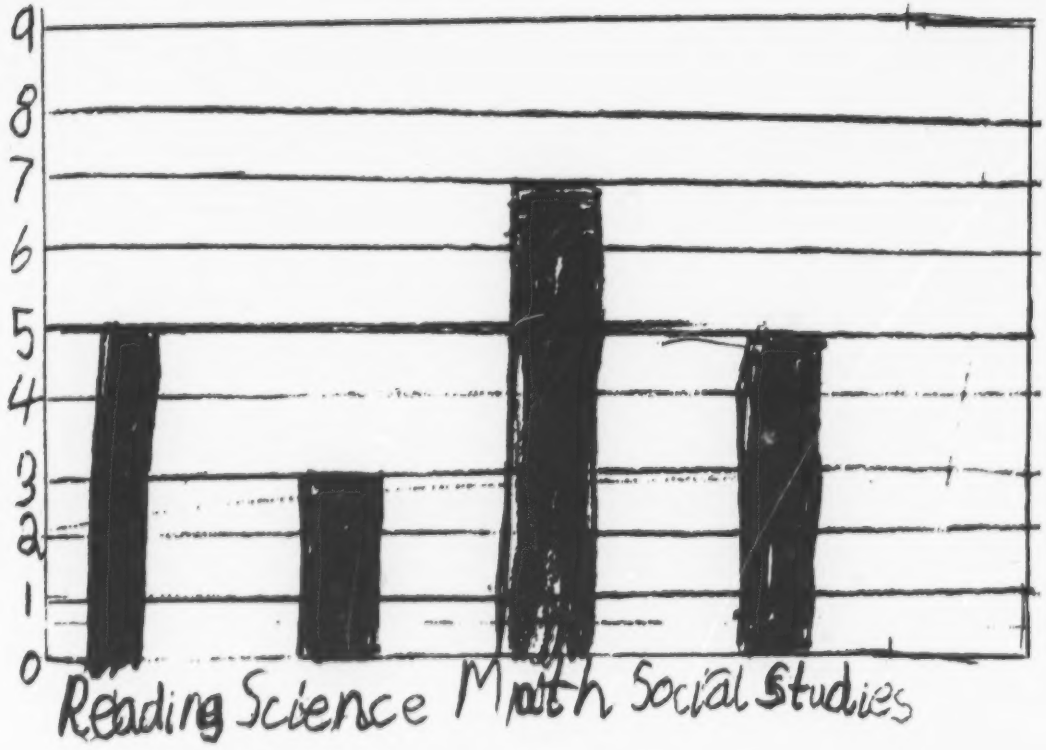
Here is what you learned.

FAVORITE SUBJECTS SURVEY

	Dion	Ben	Angie	Lacy	Kim	Scott	Nina	Kyle	Mirko	Liz
Reading	X		X				X		X	X
Science		X			X		X			
Math	X	X		X	X		X	X	X	
Social Studies		X		X		X	X			X

Go On

Use what you learned about your friends' favorite subjects to make a graph that shows how many of your friends like each subject.



Go On

Write a paragraph to explain what your graph shows.

My graph shows how much my friends like the subjects in school Math had the most people chose for it. The least people chose Science. Reading and Social Studes.

Which one of your friends likes the most subjects?

Nina

Stop

Commentary

The student was successful in constucting a graph (bar graph) in the representation of the data. The student did indicate the appropriate indicators for what each bar represents. However, additional labels to explain the meaning of each axis is needed. A graph heading should also be shown. The student indicated the maximum and minimun of the graph and was able to interpret these results.

MIDDLE SCHOOL MATHEMATICS ACTIVITY

"A New Look on a Budget" *

Task

Determine the cost of redecorating your room. You must carpet, paint two coats on the walls, and use wallpaper in some way. Draw to scale, on graph paper, each wall, including windows and doors.

Show-Me Knowledge Standards Addressed

MA 1 addition, subtraction, multiplication and division; other number sense, including numeration and estimation; and the application of these operations and concepts in the workplace and other situations

MA 2 geometric and spatial sense involving measurement (including length, area, volume), trigonometry, and similarity and transformations of shapes

Show-Me Performance Standards Addressed

1.4 use technological tools and other resources to locate, select and organize information

2.1 plan and make written, oral, and visual presentations for a variety of purposes and audiences

3.1 identify problems and define their scope and elements

3.8 assess costs, benefits, and other consequences of proposed solutions

4.1 explain reasoning and identify information used to support decisions

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Student Work

A New Look
On a Budget

Problem

Determine the cost of redecorating your room. You must carpet, paint 2 coats, and use wallpaper in some way. Draw to scale, on graph paper, each wall, including windows and doors.

Solution

After being in the same old, boring room for four years, it was time to redecorate. I had been annoying my mom about it for almost a year when she finally relented. She agreed to let me redecorate my room only under certain circumstances. She gave me a $700 budget, but I had to do all of the figuring and calling. She also added that I could keep 50% of the remaining money, if there was any! After a few days of careful thought, I decided to take on this challenge.

I began my project by measuring each wall's dimensions carefully. I labeled the walls 1, 2, 3, and 4, so I could keep track of them. I measured the woodwork, doors, and windows separately because the woodwork and doors would be a different color. I found the area of each wall by multiplying its length times height. I did the same for the woodwork, doors and windows. I kept all of this figuring in a notebook because my mind will not hold all of those numbers.

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Here is the measuring method I used: Using a yardstick, I measured the walls in yards and inches. Then I converted the yards to inches and found the total inches. To convert yards to inches, I multiplied the number of yards times 36. Next I multiplied on my calculator the length and height in inches to find the square inches of each wall. I divided the area by 144 to find the square feet. When necessary, I divided that by 9 to find the square yards. Because wall # 3 had a window, the area of the window was subtracted from the area of the wall. To find the area where I needed carpet, I multiplied the length of two walls that met at a 90 degree angle. I used the same process to find the area of the ceiling.

After doing what seemed like hours of figuring, I came up with my final figures. Wall #1 had 76 19/144 square feet of wall to be painted and 50 53/144 square feet of woodwork and doors to be painted. Wall #2 had 85 17/24 square feet of wall to be wallpapered and 5 1/24 square feet of woodwork to be painted. Wall #3 had 80 7/400 square feet of wall to be painted and 10 1/400 square feet of woodwork to be painted. Wall #4 had the same amount of square feet as wall #2 because it is parallel to wall #2 and has no extra woodwork, doors, or windows. Wall #4's 85 17/24 square feet of wall would be painted instead of wallpapered.

I went to _____ to help me with the proper amount of paint and paper to purchase. They said that 1 gallon of paint for regular walls would cover 400 square feet and 1 quart would cover 100 square feet. They also quoted the price of $14.99 for 1 gallon of wall paint tinted to be Classic Aqua. For the ceiling, I decided to use a cheaper type of

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white paint that only cost $8.99. I also selected a roll of Ocean Scene wallpaper for wall #2.

Before I could purchase the paint and wallpaper, I had to be sure how much to purchase. Under normal circumstances, the paint company would take the measurements and advise me how much to buy. My mom insisted that I do that figuring. The total amount of wall space to be painted was 241.86 square feet. The salesperson recommended two coats, which meant I actually had 483.72 square feet to paint. One gallon covers on an average 400 square feet and one quart covers 100 square feet. It was obvious that 1 gallon was not enough and 2 gallons was too much, so I purchased 1 gallon and 1 quart. This amount of paint should cover 500 square feet. The ceiling was 168.6 square feet and 1 gallon will be sufficient for two coats of paint. The wall to be papered was 85 17/24 square feet. One roll of wallpaper covers 56 square feet. Two rolls would be required, which is fine since the paper was only sold in double rolls. The wall paint cost was $14.99 for 1 gallon and $5.99 for 1 quart. The wallpaper cost $31.98

The next stop was . . . I found this store advertised in the paper. The advertised price for paint tools was cheaper than 2 other hardware stores advertised in the same paper. These same items were much more expensive at . . . . . The wood work and door surfaces to be painted were 70.45 square feet or 140.90 square feet allowing for the second coat of paint. I found I could not cover the surfaces with 1 quart and had to purchase 2 quarts. I bought 2 quarts of white woodwork paint for $11.98, a pan and paint roller kit for $5.49, and 2 paintbrushes for $4.90.

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I ordered 19 square yards of Ivory Brilliant Saxony Carpet at _____ for $16.99 a square yard. The salesperson reminded me about the closet that needed to be carpeted in the same color as the room. I had to go home, measure the closet, and then call him with the additional information. It was 1.46 square yards. Because the store sells in whole yards, not fractions, the final order was for 21 square yards of carpet, costing $356.79. This price included installation and padding

The total of all the purchases was $444.11. In Kentucky, tax is 6%. Six percent of $444.11 is $26.47. To calculate tax, I multiplied $444.11 times .06. When I added the tax, the grand total was $467.58. As stated earlier, my mom gave me a $700 budget. I was very pleased to find that I was $232.42 below budget. I received 50% of that, or $116.21. I used that to go on a class trip.

Supplies	Cost
1 gallon of Classic Aqua Paint	14.99
1 quart of Classic Aqua Paint	5.99
2 qt of white woodwork paint	11.98
1 double roll of Ocean Scene wallpaper	31.98
1 gallon of regular white paint	8.99
Pan & paint roller kit	5.49
2 paintbrushes	4.90
21 yd. of Ivory Brilliant Saxony Carpet	356.79
TOTAL	$444.11
TAX	$26.47
GRAND TOTAL	$467.58

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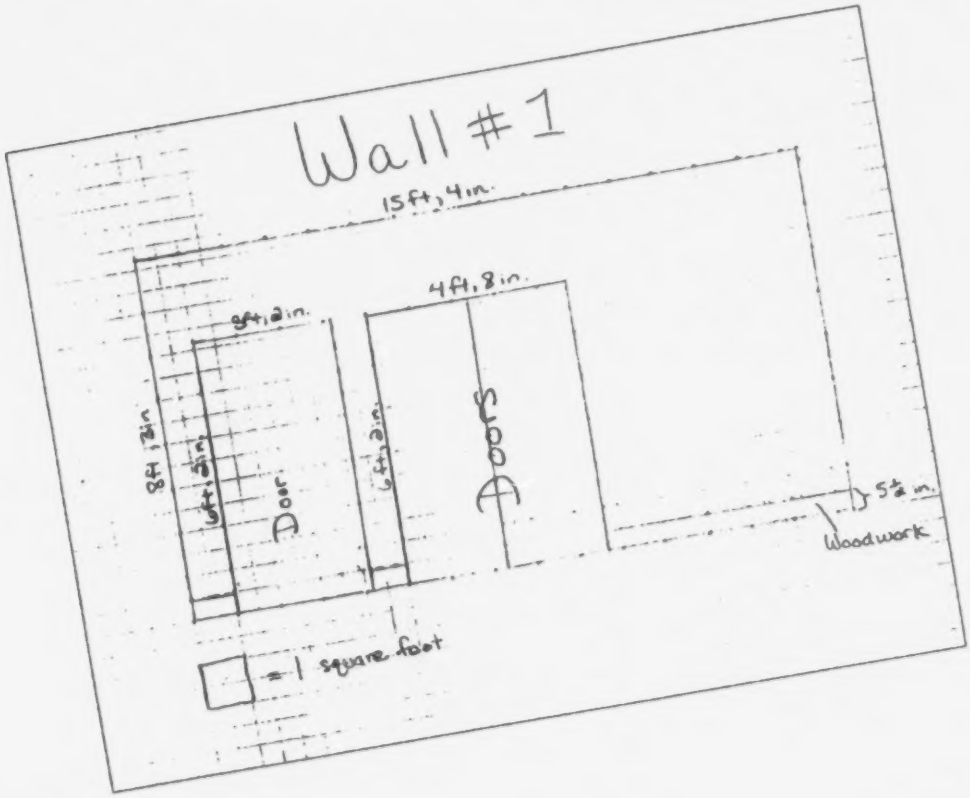

Commentary

This student work provides strong evidence for using mathematics in the workplace and other situations. The work also shows skill in problem-solving and mathematical reasoning by formulating the problem of redecoration, imposing constraints not required by the task, and determining the information needed in order to proceed. The work then shows the student implementing the redecoration, in spirit if not in fact, by determining and making appropriate measurements and calculations.

Command of the concept of area is evident, beyond simple multiplication of "length times width" of rectangles. The work also shows elements of understanding of concepts of the geometric and spatial sense standard

The many computations include manipulation of fractions and decimals and, to a lesser extent, percents. Repeatedly, the student computes with appropriate quantities and accurate measurements and calculations, providing evidence for understanding concepts of the number sense standard

Though no mention is made, it seems that the student employs a calculator when rewriting her areas as decimals instead of mixed fractions. The numbers are too precise for this kind of work, however.

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MIDDLE SCHOOL MATHEMATICS ACTIVITY

"Probability Booth" *

Task

The parent/teacher organization is having a fundraising carnival at which each grade-level class will be allocated one booth in which to conduct an activity. About 500 people are expected. For your eighth grade, design an activity that uses multiple probability events and that would attract lots of players. In the plan, please explain why you think your activity would be fun to play, how much you would charge, how much you would pay a winner, and how much profit you would expect to make.

Show-Me Knowledge Standards Addressed

MA 1 addition, subtraction, multiplication and division; other number sense, including numeration and estimation; and the application of these operations and concepts in the workplace and other situations

MA 3 data analysis, probability, and statistics

Show-Me Performance Standards Addressed

1.1 develop questions and ideas to initiate and refine research

1.4 use technological tools and other resources to locate, select and organize information

2.1 plan and make written, oral, and visual presentations for a variety of purposes and audiences

3.1 identify problems and define their scope and elements

4.1 explain reasoning and identify information used to support decisions

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Student Work

PROBABILITY BOOTH

My booth would have a coin and a pair of dice. To win you must correctly call "heads or tails" then roll an eleven or 12. I would charge $1 to play and give $10 to a winner. I feel this much of a payoff will attract many people.

I think that about 200 people would play but they would play an average of two times making a total of 400 players.

The probability of winning is 1 in 24 because it's 1 in 2 ($\frac{1}{2}$) to win the coin toss and 1 in 12 ($\frac{1}{12}$) for the roll of the dice. The roll of the dice is 1 in 12 because out of the 36 possibilities in rolling the dice 3 of them will be 11 or 12. This reduces

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PROBABILITY BOOTH

to 1/12 Then multiply 1/2 and 1/12
This becomes 1/24 Out of every
24 players one would win. My profit
out of 24 players would be $14.
With 400 players that's 16 to 17
groups of 24 for a total profit of
$384 - $408.

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Commentary

This student work provides strong evidence for statistics and probability concepts. It provides evidence for determining probabilities of events, for example, the probabilities of coin tossing and dice throwing and of the multiple event created by combining the independent events into one game. The work shows the student computing the profit he can expect at the booth, based on his assumptions about how many people will play.

In the first two paragraphs, the student formulates the problem needing analysis by deciding on a game, a fee, and a prize, and by making a guess as to the number of people who will play. He is careful to clarify and justify his claim that "P" [rolling 11 or 12] = 1/12, by alluding to the sample space of 36 equally likely outcomes when tossing two dice. The student displays understanding of expected value, realizing that his 1/24 probability implies that he can expect one winner per 24 games played.

One could challenge the assumption of 400 game players. Would that many really pay $1 for a chance to win $10 when the odds are so slim? His reasoning is clear to the "expert" reader, his teacher, though it is sketchy towards the end with unexplained statements.

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HIGH SCHOOL MATHEMATICS ACTIVITY
"Planning a Party"

Task

Students were asked to use information concerning a planned party to complete a data table and to interpret results.

Show-Me Knowledge Standards Addressed

MA 1 addition, subtraction, multiplication and division; other number sense, including numeration and estimation; and the application of these operations and concepts in the workplace and other situations

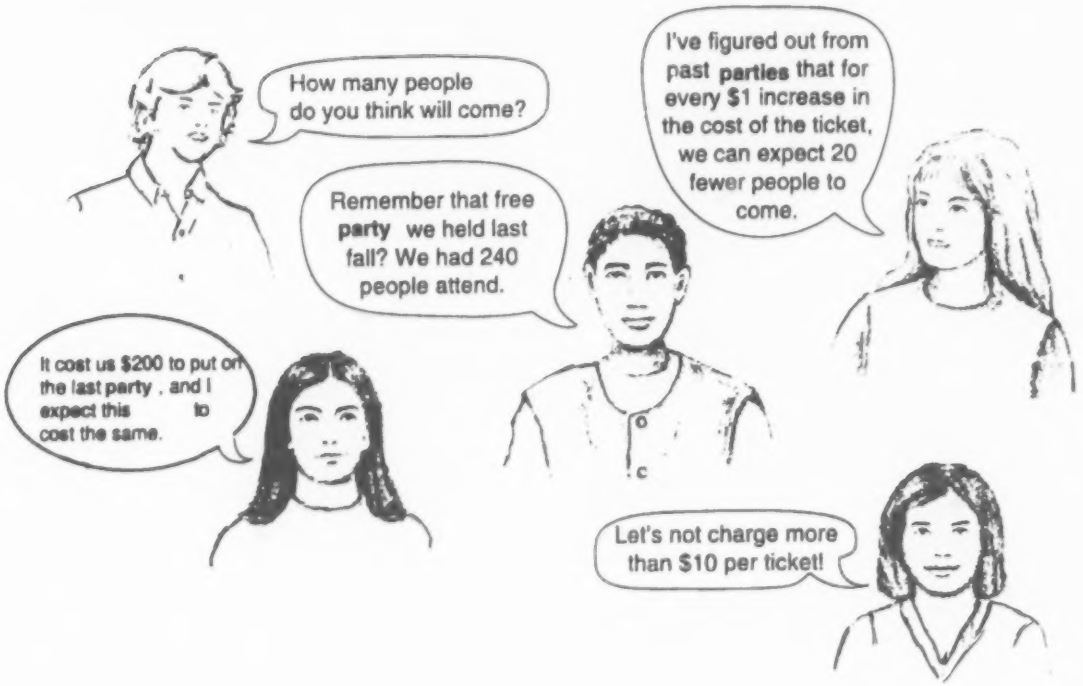
MA 3 data analysis, probability and statistics

Show-Me Performance Standards Addressed

1.6 discover and evaluate patterns and relationships in information, ideas and structures
1.8 organize data, information and ideas into useful forms (including charts, graphs, outlines) for analysis or presentation
4.1 explain reasoning and identify information used to support decisions

Student Work

Directions
Numbers 7 through 12 are about planning a party. Show all of your work and write your answers directly in this booklet.


How many people do you think will come?
I've figured out from past parties that for every $1 increase in the cost of the ticket, we can expect 20 fewer people to come.
Remember that free party we held last fall? We had 240 people attend.
It cost us $200 to put on the last party, and I expect this to cost the same.
Let's not charge more than $10 per ticket!

Use the information in the picture on page 4 to complete the table.

EXPECTED PARTY PROFITS

Cost per Ticket	Number of People Expected	Ticket Sales (in dollars)	Expected Profit or Loss
$ 0	240		$200 loss
$ 1	220	$220	$20 profit
$ 2	200	$400	$200 profit
$ 3	180	$540	$340 profit
$ 4	160	$640	$440 profit
$ 5	140	$700	$500 profit
$ 6	120	$720	$520 profit
$ 7	100	$700	$500 profit
$ 8	80	$640	$440 +
$ 9	60	$540	$340 +
$10	40	$400	$200+

540
-200

If you were to graph the expected profit or loss at each ticket price shown on this table, what would the shape of the graph be? Why would it be that shape?

because higher the tickets were fewer people came so you didn't sell as many tickets.

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School rules require a minimum of 5 chaperones at every school **party**, plus one additional chaperone for every 20 students over 100 who attend the **party**. Based on the information in the chart, what is the smallest number of chaperones that would be needed if there were no charge for admission to the **party** this year?

~~340~~ 5
~~340~~
~~+C~~
240
-100 - 5 chaperones
140
-120 1 - ch
120
-20 1
100
-20 1
80
-20 1
60
-20 1
40
-20 1
20
-20 1

12 chaperones

Commentary

The student was successful in completing the information for the table. The graph indicates a general understanding of relationships between profit and ticket prices and has indicated a maximum to the curve. Without appropriate labeling the depth of that understanding is unclear. If the domain is 1-10, the graph is inappropriately drawn. Using successive subtracting, the student was able to determine the smallest number of chaperones.

HIGH SCHOOL MATHEMATICS ACTIVITY
"Pizza Special" *

Task

Students are asked to determine profits on mini-size and medium-size pizza when given information on the costs to produce each size.

Show-Me Knowledge Standards Addressed

MA 1 addition, subtraction, multiplication and division; other number sense, including numeration and estimation; and the application of these operations and concepts in the workplace and other situations

MA 3 data analysis, probability and statistics

Show-Me Performance Standards Addressed

1.8 organize data, information and ideas into useful forms (including charts, graphs, outlines) for analysis or presentation
2.1 plan and make written, oral and visual presentations for a variety of purposes and audiences
3.4 evaluate the processes used in recognizing and solving problems

Student Work

Pizza Special

Directions

Show all of your work and write your answers directly in this booklet.

The manager of the pizza restaurant where you work wants to make pepperoni pizza the daily special. She has asked you to do a cost study to determine the profits the restaurant will make if it offers the mini-size and the medium-size pepperoni pizza at the daily special prices.

It costs a total of $0.70 for the cheese, sauce, and dough for one mini-pizza and $2.50 for one medium pizza.
We put 4 slices of pepperoni on each mini-pizza and 16 slices on each medium pizza.
We slice a 24-inch stick of pepperoni into pieces that are $\frac{1}{16}$-inch thick. One stick costs $9.99.

Use the information provided on page 10 to prepare a report of your findings for your manager. Write your report in the space provided on page 12. Include in your report:

- A summary of your findings that states how much the restaurant will make from the sale of each size of pepperoni pizza
- A written description of each step of the process you use to reach your conclusions

Use the space below to calculate the profit made from the sale of a mini-size pizza and the profit made from the sale of a medium size pizza. Show all of your work.

16 pieces in an inch

24

384 pieces in a pie for 9.99

24
9

96 | 9.99

21 | 9.99 .42

Mini	Medium
70¢ 10¢ 80¢ to make	2.50 42¢ 2.92 to make

3.99
- .80
$3.19 profit

11.99
- 2.92
$9.07 profit

REPORT OF FINDINGS

Sir I have found that, that is a very good idea. You will make #3.12 off of 1 mini pizza or $9.07 off of a large. 16 pieces of pepperoni in every inch, so that is enough for 24 medium or 96 mini pieces in a sack. I divided 9.99 by those 2 #'s and got how much pepperoni costs for 1 piece. I already knew how much other stuff costed and just added them together. Then subtracted them from how much you charge and got your profit.

STOP

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Commentary

The student correctly calculated the profits for each size pizza. Most of the work was shown but was somewhat disorganized. The table showed the total costs for each pizza size followed by subtraction of costs to show profits. The written finding provided an adequate explanation of the process used.

Dear Users of *Missouri's Framework for Curriculum Development in*_____:
(subject)

We hope that this framework has been helpful to you in developing local curriculum aligned with The Show-Me Standards. Please take a few minutes to give the Department of Elementary and Secondary Education comments concerning your use of this document.

1. How did you use this framework? (Explain process.)

2. What parts of the framework were most helpful and why?

3. What suggestions do you have that would help make future versions of this document more useful?

4. If you have specific suggestions concerning certain pages, please indicate those page numbers and explain. Use the back of the page, if necessary.

Important Note:

By the end of the 1996-97 school year, we would also like to publicize Missouri students' quality work at elementary, middle and high school levels related to the Show-Me Standards. As you and other teachers in your district continue to develop performance-based activities/ assessments, please collect samples of quality student work. Samples submitted to DESE should follow the format in Appendix A of the framework.

Please return this sheet with your name and address to the Department of Elementary and Secondary Education, Curriculum Services, PO Box 480, Jefferson City, MO 65102. Call 573-751-2625 for more information concerning submissions of quality student work.

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